## Applications

## TEACH

Exercises 2, 3, 7, and 9 These exercises are related. Have students first identify which are future values of periodic investment and which are present values of periodic investment problems.

## Exercise 4

Here, students first have to calculate the future value of a single deposit investment in order to determine the amount from which the withdrawal will be made.

## Exercise 5

This is a good guided discovery activity to use during class. Students can work in small groups through each step.

## ANSWERS

1. Retirement requires advanced planning and preparation. A retirement savings plan requires years to grow. Mr. Cook warns all of us (young and old) that without advanced planning for retirement, we could face years of poverty rather than a comfortable old age.

> A whole generation of Americans will retire in poverty instead of prosperity because they simply are not preparing for retirement now.
-Scott Cook, American businessman

1. Explain how the quote can be interpreted. See margin.
2. Ricky is 35 years old. He plans to retire when he is 63 . He has opened a traditional retirement account that pays $1 \%$ interest compounded monthly. If he makes monthly deposits of $\$ 400$, how much will he have in the account by the time he retires? $\$ 155,028.26$
3. Jay just graduated from college and he has decided to open a retirement account that pays $1.75 \%$ interest compounded monthly. If he has direct deposits of $\$ 100$ per month taken out of his paycheck, how much will he have in the account after 42 years? $\$ 74,356.50$
4. At the age of 30 , Jasmine started a retirement account with $\$ 50,000$, which compounded interest semi-annually with an APR of $1.75 \%$. She made no further deposits. After 25 years, she decided to withdraw $50 \%$ of what had accumulated in the account so that she could contribute toward her grandchild's college education. She had to pay a $10 \%$ penalty on the early withdrawal. What was her penalty? $\$ 3864.72$
5. A taxpayer who pays $22 \%$ in taxes each year has these two accounts.

Account $1: \$ 10,000$ is placed in a tax-deferred account that pays $2.1 \%$ interest compounded annually for 25 years.
Account 2: $\$ 10,000$ is placed in a taxable account that pays $2.1 \%$ interest compounded annually for 25 years.
a. How much is in Account 1 after the 25 -year period? \$16,812.94
b. Since the taxpayer pays $22 \%$ of all income in taxes, $22 \%$ of the interest he makes each year will go toward taxes. Therefore, his annual interest rate in actuality is $22 \%$ less than the $2.1 \%$ quoted rate. What is his real annual interest rate? $1.638 \%$
c. How much will he actually have made after the 25 -year period in Account 2 if taxes are taken into consideration? The account balance will be $\$ 15,010.76$. He will have made $\$ 5,010.76$ in interest.
6. Laura has been contributing to a retirement account that pays $1.75 \%$ interest with pre-tax dollars. This account compounds interest monthly. She has put $\$ 500$ per month into the account. At the end of 10 years, she needed to pay some medical bills and had to withdraw $15 \%$ of the money that was in the account.
a. Rounded to the nearest dollar, how much did she withdraw? $\$ 9,828$
b. Laura pays $23 \%$ of her income in taxes. What was her tax on the amount of the withdrawal (rounded to the nearest dollar)? $\$ 2,260$
c. She had to pay a $10 \%$ early withdrawal penalty. How much was she required to pay, rounded to the nearest dollar? \$983
7. Fiona opened a retirement account that has an annual rate of $2.2 \%$. She is planning on retiring in 20 years. How much must she deposit into that account each year so that she can have a total of $\$ 600,000$ by the time she retires? \$24,206.05
8. John is 60 years old. He plans to retire in 2 years. He now has $\$ 400,000$ in a savings account that yields $2.1 \%$ interest compounded monthly. He has calculated that his final working year's salary will be $\$ 88,000$. He has been told by his financial advisor that he should have $60-70 \%$ of his final year's annual income available for use each year when he retires.
a. What is the range of income that his financial advisor thinks he must have per year once he retires? $\$ 52,800-\$ 61,600$
b. Determine how much he will have in his account at the ages of 61 and 62. (margin)
c. Assume that John is planning on using $65 \%$ of his current salary in each of his first 5 years of retirement. What should that annual amount be? $\$ 57,200$
d. John has decided that he will need $\$ 2,000$ each month from his savings account to help him reach his desired annual income during retirement. Will John be able to make withdrawals of 2,000 each month from his savings account for 20 years? Explain your reasoning. See margin.
9. Bob can afford to deposit $\$ 400$ a month into a retirement account that compounds interest monthly with an APR of $1.8 \%$. His plan is to have $\$ 200,000$ saved so that he can then retire. Approximately how long will it take him to reach this goal? 31 years
10. Jack contributed $\$ 400$ per month into his retirement account in pre-tax dollars during the last tax year. His taxable income for the year was $\$ 62,350$.
He files taxes as a single taxpayer.
a. What would his taxable income have been had he contributed to the account in after-tax dollars? $\$ 67,150$
b. Use the tax table below to calculate his tax in both the pre-tax and after-tax contribution situations. Pre-tax: 11,388; After-tax: \$12,588
c. How much did Jack save in taxes during that year? \$1,200

| If line 43 (taxable income) is- |  | And you are - |  |  |  | If line 43 (taxable income) is- |  | And you are - |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| At least | But <br> less <br> than | Single | Married filing jointly* <br> Your tax | Married <br> filing <br> sepa- <br> rately <br> $x$ is - | Head of a household | At least | But <br> less <br> than | Single | Married <br> filing jointly* <br> Your ta | Married <br> filing <br> sepa- <br> rately <br> $x$ is - | Head of a household |
| 62,000 |  |  |  |  |  | 67,000 |  |  |  |  |  |
| 62,000 | 62,050 | 11,300 | 8,381 | 11,300 | 9,829 | 67,000 | 67,050 | 12,550 | 9,131 | 12,550 | 11.079 |
| 62,050 | 62,100 | 11,313 | 8.389 | 11,313 | 9,841 | 67,050 | 67,100 | 12,563 | 9,139 | 12,563 | 11,091 |
| 62,100 | 62,150 | 11,325 | 8.396 | 11,325 | 9,854 | 67,100 | 67,150 | 12,575 | 9,145 | 12,575 | 11,104 |
| 62,150 | 62,200 | 11,338 | 8,404 | 11,338 | 9,866 | 67,150 | 67,200 | 12,588 | 9,154 | 12,588 | 11,116 |
| 62,200 | 62,250 | 11,350 | 8.411 | 11,350 | 9.879 | 67,200 | 67,250 | 12,600 | 9,161 | 12,600 | 11,129 |
| 62,250 | 62,300 | 11,363 | 8,419 | 11,363 | 9,891 | 67,250 | 67,300 | 12,613 | 9,169 | 12,613 | 11,141 |
| 62,300 | 62,350 | 11,375 | 8,426 | 11,375 | 9,904 | 67,300 | 67,350 | 12,625 | 9,176 | 12,625 | 11,154 |
| 62,350 | 62,400 | 11,388 | 8,434 | 11,388 | 9,916 | 67,350 | 67,400 | 12,638 | 9.184 | 12,638 | 11,166 |
| 62,400 | 62,450 | 11,400 | 8,441 | 11,400 | 9,929 | 67,400 | 67,450 | 12,650 | 9,191 | 12,650 | 11,179 |
| 62,450 | 62,500 | 11,413 | 8,449 | 11,413 | 9,941 | 67,450 | 67,500 | 12,663 | 9.199 | 12,663 | 11,191 |
| 62,500 | 62,550 | 11,425 | 8,456 | 11,425 | 9,954 | 67,500 | 67,550 | 12,675 | 9,206 | 12,675 | 11,204 |
| 62,550 | 62,600 | 11,438 | 8,464 | 11,438 | 9,966 | 67,550 | 67,600 | 12,688 | 9,214 | 12,688 | 11,216 |
| 62,600 | 62,650 | 11,450 | 8,471 | 11,450 | 9,979 | 67,600 | 67,650 | 12,700 | 9,221 | 12,700 | 11,229 |
| $\mathbf{6 2 , 6 5 0}$ $\mathbf{6 2 , 7 0 0}$ | $\mathbf{6 2 , 7 0 0}$ $\mathbf{6 2 , 7 5 0}$ | 11,463 11,475 | 8.479 8.486 | 11,463 | 9,991 | 67,650 | 67,700 | 12,713 | 9.229 | 12,713 | 11,241 |
| 62,700 | 62,750 | 11,475 | 8.486 | 11,475 | 10,004 | 67,700 | 67,750 | 12,725 | 9.236 | 12,725 | 11,254 |
| 62,750 | 62,800 | 11,488 | 8,494 | 11,488 | 10,016 | 67,750 | 67,800 | 12,738 | 9.244 | 12,738 |  |
| 62,800 | 62,850 | 11,500 | 8,501 | 11,500 | 10,029 | 67,800 | 67,850 | 12,750 | 9.251 | 12,750 | 11,279 |
| 62,850 | 62,900 | 11,513 | 8.509 | 11,513 | 10,041 | 67,850 | 67,900 | 12,763 | 9,259 | 12,763 | 11,291 |
| 62,900 | 62,950 | 11,525 | 8,516 | 11,525 | 10,054 | 67,900 | 67,950 | 12,775 | 9,266 | 12,775 | 11,304 |
| 62,950 | 63,000 | 11,538 | 8,524 | 11,538 | 10,066 | 67,950 | 68,000 | 12,788 | 9,274 | 12,788 | 11,316 |

## TEACH

## Exercise 8

Before assigning part (d) of this exercise, encourage students to go back to Section 2-10 and review the lesson on systematic withdrawal accounts. There are two ways to view this problem. Use the balance in the account when John is 62 years old from 8b. Then use the systematic withdrawal formula to determine how long it will take for monthly withdrawals of $\$ 2000$ to deplete the entire account. You will see that the answer is over 20 years. This method will require the use of logs. The other way to find the answer is to use the same systematic withdrawal formula and replace $t$ with 20 . The result will yield the amount that has been withdrawn from the account after 20 years. Since this number is less than the principal, John can be assured that he will have enough in the account to make the withdrawals.

## Exercise 9

Since the problem asks for a length of time, and time is an exponent, this problem will require logs in the solution.

## ANSWERS

8b. $\$ 408,481.32$ and $\$ 417,142.48$ respectively
8 d . The amount in the account at retirement age of 62 will last a little over 21.5 years.

## TEACH

## Exercise 11

If you anticipate some students experiencing difficulty with the algebraic expressions in this exercise, give them a similar problem with numbers first. Once completed, ask them to do Exercise 11 using the numerical problem as a guide.

## Exercises 13 and 14

Be sure to assign (and possibly review) Exercise 13 before having students work on Exercise 14. They are related.

## ANSWERS

Exercise 11c. $\frac{10.03 x t}{100}+1.003 x$
11. Mark is an accountant who has been contributing to his retirement account for the last 15 years with pre-tax dollars. The account compounds interest semi-annually at a rate of $2.25 \%$. He contributes $x$ dollars after each 6 -montil period, and this has not changed over the life of the account.
a. How much will he have in the account after 20 years of saving? Round numbers to the nearest hundredth. 50.17x
b. After 20 years of contributions, he needed to withdraw $20 \%$ of the moner in his account to pay for his children's education. Write an expression far the withdrawal amount. 10.03x
c. Mark pays $t$ percent of his income in taxes. Write an algebraic expression for the combined total of his tax and the $10 \%$ early withdrawal penalty
12. Jhanvi is a 40 -year-old executive for a department store. She files taxes as head of household. She needed to withdraw $\$ 45,000$ from her tax-deferred retirement account to put a down payment on a new condominium. Jhannis taxable income for that year was $\$ 110,550$, excluding the $\$ 45,000$ early withdrawal from her retirement account.
a. Use the tax computation worksheet shown below to calculate Jhanvi's tat had she not made the early withdrawal. \$21,960

Section D-Use if your filing status is Head of houshold. Complete the row below that applies to you.

| Taxable income If line 43 is- | (a) <br> Enter the amount from line 43 | (b) <br> Multiplication amount | (c) Multiply (a) by (b) | (d) <br> Subtraction amount | Tax <br> Subtract (d) from Enter the result here ant on Form 1040, line - |
| :---: | :---: | :---: | :---: | :---: | :---: |
| At least $\$ 100,000$ but not over \$129,600 | \$ | x $25 \%(0.25)$ | \$ | \$5,677.50 | \$ |
| Over $\$ 129,600$ but not over \$209,850 | \$ | $\times 28 \%(0.28)$ | \$ | \$9,565.50 | \$ |
| Over \$209,850 but not over \$411,500 | \$ | $\times 33 \%(0.33)$ | \$ | \$20,058.00 | \$ |
| Over $\$ 411,500$ but not over \$439,000 | \$ | $\times 35 \%(0.35)$ | \$ | \$28,288.00 | \$ |
| Over \$439,000 | \$ | $\times 39.6 \%$ (0.396) | \$ | \$48,482.00 | \$ |

b. Use the same worksheet to calculate her tax with an increase in her taxable income of $\$ 45,000$, which represents the early withdrawal. $\$ 33,988.50$
c. How much more in taxes did she pay because of the early withdrawal?
d. If Jhanvi paid a $10 \%$ early withdrawal fee, how much was her early withdrawal penalty? $\$ 4,500$
13. Nelson makes $\$ 120,000$ per year. His employer offers a $401(\mathrm{k})$ plan in which they will match $40 \%$ of his contributions up to a maximum of $7 \%$ of his annual salary. His employer allows contributions up to a maximum of $15 \%$ of Nelson's salary per year. If Nelson contributes $\$ 200$ out of each biweekly paycheck, how much will his employer contribute to his $401(\mathrm{k})$ ? $\$ 2,080$
14. Mike makes $Y$ dollars per year. His company offers a matching retirement plan in which they agree to match $M$ percent of his contributions up to $P$ percent of his salary. Write an algebraic expression for the maximum value of the employer's matching contribution. $\frac{(\text { YMP })}{10000}$

