

9-2 Objectives

- ▼ To graph inverse variations (p. 485)
- ▼ To graph translations of inverse variations (p. 487)

The graph of an inverse variation has two parts called **branches**. The graph of $y = \frac{k}{x-b} + c$ is a translation of $y = \frac{k}{x}$ by b units horizontally and c units vertically. It has a vertical asymptote at $x = b$ and a horizontal asymptote at $y = c$.

Sketch the graph of each equation. 18-21. See margin.

18. $y = \frac{1}{x}$ 19. $y = \frac{-2}{x^2}$ 20. $y = \frac{-1}{x} - 4$ 21. $y = \frac{1}{x-2} + 3$

Write an equation for the translation of $xy = 4$ that has the given asymptotes.

22. $x = 0, y = 3$ 23. $x = 2, y = 2$ 24. $x = -3, y = 1$
 $y = \frac{4}{x} + 3$ $y = \frac{4}{x-2} + 2$ $y = \frac{4}{x+3} - 1$

9-3 Objectives

- ▼ To identify properties of rational functions (p. 491)
- ▼ To graph rational functions (p. 494)

The rational function $f(x) = \frac{P(x)}{Q(x)}$ has a **point of discontinuity** for each real number a such that $Q(a) = 0$. If $P(x)$ and $Q(x)$ have no common factors, then the graph of $f(x)$ has a vertical asymptote when $Q(x) = 0$. If $P(x)$ and $Q(x)$ have a common factor $(x-a)$, then there is a hole or a vertical asymptote at $x = a$.

If the degree of $Q(x)$ is greater than the degree of $P(x)$, then the graph of $f(x)$ has a horizontal asymptote at $y = 0$.

If $P(x)$ and $Q(x)$ have equal degrees, then there is a horizontal asymptote at $y = \frac{a}{b}$, where a and b are the coefficients of the terms of greatest degree in $P(x)$ and $Q(x)$.

If the degree of $P(x)$ is greater than the degree of $Q(x)$, then there is no horizontal asymptote.

25. A headset can be manufactured for \$17. The development cost is \$1500. Graph the function that represents the average cost of a headset. About how many must be manufactured to result in a cost of less than \$5 per headset? See margin.

30. $\frac{(x-1)(x+1)}{x+3}$;
 $x \neq -4, -3, \text{ or } 6$

31. $\frac{(2x-1)(x+1)}{x+4}$;
 $x \neq -4, -1, \text{ or } 0$

Find any points of discontinuity for each rational function. Sketch the graph. Describe any vertical or horizontal asymptotes and any holes. 26-29. See margin.

26. $y = \frac{2.5}{x+7}$ 27. $y = \frac{x-1}{(x+2)(x-1)}$
 28. $y = \frac{x^3-1}{x^2-1}$ 29. $y = \frac{2x^2+3}{x^2+2}$

9-4 and 9-5 Objectives

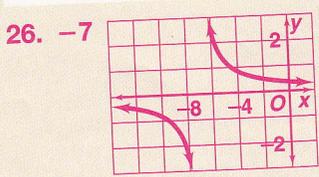
- ▼ To simplify rational expressions (p. 499)
- ▼ To multiply and divide rational expressions (p. 500)
- ▼ To add and subtract rational expressions (p. 504)
- ▼ To simplify complex fractions (p. 506)

A rational expression is in **simplest form** when its numerator and denominator are polynomials that have no common divisors. To add or subtract rational expressions with different denominators, write each expression with the common denominator.

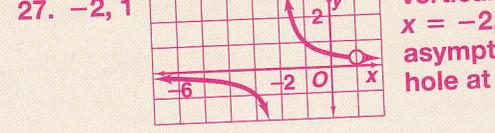
A fraction that has a fraction in its numerator or denominator or in both is a **complex fraction**. You can simplify a complex fraction by multiplying the numerator and denominator by the LCD of all the rational expressions.

Simplify each rational expression. State any restrictions on the variable. 30-31. See above left.

30. $\frac{x^2-2x-24}{x^2+7x+12} \cdot \frac{x^2-1}{x-6}$ 31. $\frac{4x^2-2x}{x^2+5x+4} \div \frac{2x}{x^2+2x}$



vertical asymptote $x = -7$, horizontal asymptote $y = 0$



vertical asymptote $x = -2$, asymptote hole at $x = 1$

Need Help?

$V = \frac{4}{3}\pi r^3$
 $S.A. = 4\pi r^2$
 $\frac{1}{a} = \frac{1}{d_i} + \frac{1}{d_o}$

32. What is the ratio of the volume of a sphere to its surface area? $\frac{r}{3}$
33. A camera's focal length is 4 cm. If the lens is 6 cm from the film, what is the distance from the lens to an object that is in focus? **12 cm**

Simplify each expression.

34. $\frac{3x}{x^2 - 4} + \frac{6}{x + 2} = \frac{3(3x - 4)}{(x - 2)(x + 2)}$
 35. $\frac{1}{x^2 - 1} - \frac{2}{x^2 + 3x} = \frac{-x^2 + 3x + 2}{x(x + 1)(x - 1)(x + 3)}$
 36. $\frac{2 - \frac{2}{3}}{3 - \frac{1}{2}} = \frac{8}{15}$
 37. $\frac{\frac{1}{x + y}}{\frac{4}{4(x + y)}} = \frac{1}{4(x + y)}$

Solving a rational equation often requires multiplying both sides by an algebraic expression. This may introduce an extraneous solution—a solution of a derived equation but not of the original equation. Check all possible solutions in the original equation.

Solve each equation. Check each solution.

38. $\frac{1}{x} = \frac{5}{x - 4} \quad -1$
 39. $\frac{2}{x + 3} - \frac{1}{x} = \frac{-6}{x(x + 3)} \quad \text{no solution}$
 40. $\frac{1}{2} + \frac{x}{6} = \frac{18}{x} \quad -12, 9$

41. One pump can fill a water cistern twice as fast as a second pump. Working together, the two pumps can fill the cistern in 5 hours. Find how long it takes each pump to fill the cistern when working alone. **7.5 h, 15 h**

When the occurrence of one event affects the probability of another event, the two events are **dependent events**. When the occurrence of one event does not affect the probability of another event, the two events are **independent events**. When two events cannot happen at the same time, the events are **mutually exclusive events**.

If A and B are independent, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

If A and B are mutually exclusive, then $P(A \text{ and } B) = 0$.

If A and B are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$.

If A and B are not mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Classify each pair of events as dependent or independent.

42. A student in your algebra class is selected at random. One of the remaining students is selected at random. **dependent**
43. A number 1 through 6 is chosen by tossing a standard number cube. The same number cube is tossed again to select a number 1 through 6. **independent**

Two standard number cubes are tossed. State whether the events are mutually exclusive. not mutually exclusive

44. One of the numbers is 1 less than the other. The sum is odd.
45. The sum is greater than 10. Six is one of the numbers. **not mutually exclusive**

A standard number cube is tossed. Find each probability.

46. a 5 or a 6 $\frac{1}{3}$ 47. an even number or a number greater than 4 $\frac{2}{3}$
48. an odd number or a number less than or equal to 5 $\frac{5}{6}$

Objectives

To solve rational equations (p. 512)

To use rational equations in solving problems (p. 513)

Objectives

To find the probabilities of events A and B (p. 519)

To find the probabilities of events A or B (p. 521)

Alternative Assessment, Form C

Alternative Assessment Form C

Chapter 9

Give complete answers.

- TASK 1**
- Write a function that shows an inverse variation situation.
 - Find the constant of the inverse variation.
 - Determine the dependent and independent variables.
 - Identify the domain and range.
 - Find the values of any asymptotes.
 - Graph the function, making sure to indicate any asymptotes.

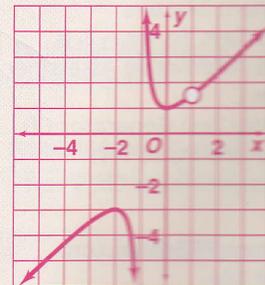
TASK 2

Use the function you wrote in Task 1 to answer each question.

- Let the dependent variable be 3. Find the value of the independent variable.
- Let the independent variable be 6. Find the value of the dependent variable.
- What value does the dependent variable approach when the independent variable approaches infinity?
- What value does the independent variable approach when the dependent variable approaches infinity?

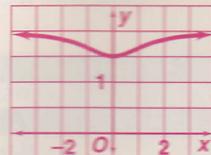
Algebra 2 Chapter 9

28. 1, -1



vertical asymptote $x = -1$, hole at $x = 1$

29. no points of discontinuity



horizontal asymptote $y = 2$

Resources

- Teaching Resources**
 - Ch. 9 Test, Forms A & B
 - Ch. 9 Alternative Assessment, Form C
- Reaching All Students**
 - Spanish Ch. 9 Test, Forms A & B
 - Spanish Ch. 9 Alternative Assessment, Form C

ASSESSMENT SYSTEM

- Assessment Masters**
 - Ch. 9 Test, Forms A & B
 - Ch. 9 Alternative Assessment, Form C
- Computer Test Generator CD**
 - Ch. 9 pre-made Test
 - Make your own Ch. 9 test

www.PHSchool.com

- Student Site**
 - Self-grading Chapter 9 Test
- Teacher Center**
 - Resources

Plus **iTEXT**

Chapter Test — Form B

Chapter Test — Form A

Chapter Test Form A Chapter 9

Write a function that models each variation.

- $x = -1$ when $y = 5$; y varies inversely with x .
- $x = 3$ and $y = 12$ when $z = 2$; z varies directly with y and inversely with x .

In the relationship between the values in each table a direct variation, an inverse variation, or neither? Write an equation to model any direct or inverse variation.

- | | | | |
|---|----|----|-----|
| x | -2 | 4 | 8 |
| y | 4 | -8 | -12 |
- | | | | |
|---|----|----|---|
| x | -2 | -1 | 3 |
| y | 4 | -1 | 4 |

Write an equation for the translation of $y = \frac{2}{3}$ with the given asymptotes.

- $x = 1$; $y = -1$
- $x = 5$; $y = \frac{1}{2}$

For each rational function, identify any holes or horizontal or vertical asymptotes of its graph.

- $y = \frac{x-2}{(x+1)(x-2)}$
- $y = \frac{2x^2}{x^2 - 4x}$
- $y = \frac{1}{x+2} - 3$
- $y = \frac{5}{x-2} + 1$
- $y = \frac{x^2 + 5}{x - 5}$
- $y = \frac{x+2}{(x+2)(x-3)}$

9–16. See margin.

Simplify each rational expression. State any restrictions on the variable.

- $\frac{x^2 + 7x + 12}{x^2 - 9}$ $\frac{x+4}{x-3}$; $x \neq -3$ or 3
- $\frac{(x+3)(2x-1)}{x(x+4)} \div \frac{(-x-3)(2x+1)}{x}$ **See back of book.**

Write a function that models each variation. $y = \frac{-16}{x}$

- $x = 2$ when $y = -8$. y varies inversely with x .
- $x = 0.2$ and $y = 3$ when $z = 2$. z varies jointly with x and y . $z = \frac{10}{3}xy$
- $x = \frac{1}{3}$, $y = \frac{1}{5}$, and $r = 3$ when $z = \frac{1}{2}$. z varies directly with x and inversely with the product of r^2 and y . $z = \frac{2.7x}{r^2y}$

Is the relationship between the values in each table a direct variation, an inverse variation, or neither? Write an equation to model any direct or inverse variation.

4.

x	3	5	7	9
y	6	8	10	12

neither

5.

x	4	6	8	10
y	10	8	6	4

neither

6.

x	4	8	16	32
y	32	16	8	4

inverse variation; $y = \frac{128}{x}$

Graph the translation of $y = \frac{7}{x}$ with the given asymptotes. Write the equation of the translation.

- $x = 1$; $y = 2$
- $x = -3$; $y = -2$

7–8. See back of book.

For each rational function, identify any holes, or horizontal or vertical asymptotes of its graph.

- $y = \frac{x+1}{x-1}$
- $y = \frac{x+3}{x+3}$
- $y = \frac{x-2}{(x+1)(x-2)}$
- $y = \frac{2x^2}{x^2 - 4x}$
- $y = \frac{1}{x+2} - 3$
- $y = \frac{5}{x-2} + 1$
- $y = \frac{x^2 + 5}{x - 5}$
- $y = \frac{x+2}{(x+2)(x-3)}$

9–16. See margin. Simplify each rational expression. State any restrictions on the variable.

- $\frac{x^2 + 7x + 12}{x^2 - 9}$ $\frac{x+4}{x-3}$; $x \neq -3$ or 3
- $\frac{(x+3)(2x-1)}{x(x+4)} \div \frac{(-x-3)(2x+1)}{x}$ **See back of book.**

19. Open-Ended Write a function whose graph has a vertical asymptote, a horizontal asymptote, and a hole. **Check students' work.**

Find the least common multiple of each pair of polynomials.

- $3x + 5$ and $9x^2 - 25$ **$9x^2 - 25$**
- $5(x+3)(x+1)$ and $2(x+1)(x-3)$ **$10(x+3)(x+1)(x-3)$**

Simplify each sum or difference.

- $\frac{x+2}{(x-3)(x+1)} + \frac{(x-1)(x+2)}{x-3}$ $\frac{x^2(x+2)}{(x-3)(x+1)}$
- $\frac{x^2-1}{(x-2)(3x-1)} - \frac{x+1}{x+3}$ $-\frac{(x+1)(2x^2-9)}{(x-2)(3x-1)}$
- $\frac{x(x+4)}{x-2} + \frac{x-1}{x^2-4}$ $\frac{x^3+6x^2+9x-1}{x^2-4}$

Simplify each complex fraction.

- $\frac{\frac{2}{x}}{1-\frac{1}{y}}$ $\frac{2y}{x(y-1)}$
- $\frac{3-\frac{3}{4}}{\frac{1}{2}-\frac{1}{4}}$ **9**

Solve each equation. Check each solution.

- $\frac{x}{2} = \frac{x+1}{4}$ **1**
- $\frac{3x}{x+1} = 0$ **0**
- $\frac{1}{x} + \frac{1}{3} = \frac{6}{x^2}$ **-6 or 3**
- $\frac{3}{x-1} = \frac{4}{3x+2}$ **28.**
- $\frac{3}{x+1} = \frac{1}{x^2-1}$ **30.**
- $\frac{1}{x} + \frac{x}{x+2} = 1$ **32.**

33. Almir can seal a driveway in 4 hours. Working together, he and Louis can seal it in 2.3 hours. How long would it take Louis to seal it working alone?

Two standard number cubes are tossed. State whether the events are mutually exclusive. Then find $P(A \text{ and } B)$.

- A means their sum is 12; B means both are 6. **mutually exclusive**
- A means they are equal; B means their sum is a multiple of 3. **not mutually exclusive; $\frac{4}{9}$**
- a. Writing** Suppose you select a number at random from the set $\{90, 91, 92, \dots, 99\}$. Event A is selecting a multiple of 3. Event B is selecting a multiple of 4. Are these two events mutually exclusive? Explain. **a. No; 96 is multiple of 3 and 4.**
- Find $P(A \text{ and } B)$. **$\frac{1}{10}$**
- Find $P(A \text{ or } B)$. **$\frac{1}{2}$**

- vertical asymptote $x = 1$, horizontal asymptote $y = 1$
- hole at $x = -3$

- hole at $x = 2$, vertical asymptote $x = -1$, horizontal asymptote $y = 0$
- hole at $x = 0$, vertical asymptote $x = 4$, horizontal asymptote $y = 2$

- vertical asymptote $x = -2$, horizontal asymptote $y = -3$
- vertical asymptote $x = 2$, horizontal asymptote $y = 1$

- vertical asymptote $x = 3$; horizontal asymptote $y = 0$