

3. Practice

EXERCISES

For more practice, see *Extra Practice*

Practice and Problem Solving

Assignment Guide

1 Objective

- A B** Core 1–9, 18–23, 26–28
- C** Extension 32

2 Objective

- A B** Core 10–17, 24–25, 29–31
- C** Extension 33–34

Standardized Test Prep 35–38

Mixed Review 39–44

Error Prevention

Exercises 18–22 Suggest students first make sure the matrices have the same dimension.

Enrichment 4-2

Reteaching 4-2

Practice 4-2

Practice 4-2 Adding and Subtracting Matrices

Find the value of each variable.

- $\begin{bmatrix} 4 & -2 & 20 \\ -1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 5 & -1 \\ 10 & 30 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 7 & 4 \\ 2 & 0 & 3 & 3 \end{bmatrix}$
- $\begin{bmatrix} 3 & 5 & -2 & 3 \\ 2 & 0 & 3 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 7 & 4 \\ 2 & 0 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 7 & 4 \\ 2 & 0 & 3 & 3 \end{bmatrix}$
- $\begin{bmatrix} 15 & 7 \\ 10 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -11 \\ -10 & -10 \end{bmatrix} = \begin{bmatrix} 24 & 1 \\ -4 & 25 \end{bmatrix} - x$

Use the information in the table.

4. Put the data in two matrices one for males and one for females.

	1967–1987		2007–2009	
	Males	Females	Males	Females
Beta	37	23	56	58
Spanish	0	93	76	82
Cheer	67	0	102	74
Library	4	18	27	29

5. Use matrix subtraction to find the difference between the number of males and the number of females in each club each year.

Find each sum or difference.

- $\begin{bmatrix} 1 & -2 \\ -3 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$
- $\begin{bmatrix} 8 & -5 & -5 \\ 4 & -8 & 10 \\ 3 & -18 & -15 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 \\ 1 & -2 & -3 \\ 2 & -3 & 3 \end{bmatrix}$
- $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} -2 & -2 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$
- $\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} + \begin{bmatrix} -2 & -2 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$

Solve each matrix equation.

- $X + \begin{bmatrix} 20 & -6 & -5 \\ 19 & -2 & -5 \\ -1 & 0 & -4 \end{bmatrix} = \begin{bmatrix} -7 & 92 & -5 \\ 0 & 91 & -6 \\ -1 & 0 & 12 \end{bmatrix}$
- $\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} + X = \begin{bmatrix} 2 & -3 \\ 3 & -3 \end{bmatrix} - X$

Determine whether the two matrices in each pair are equal. Justify your reasoning.

- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$
- $\begin{bmatrix} 331 & 315 \\ 7 & 32 \end{bmatrix}$ and $\begin{bmatrix} 3 & 42 \\ 7 & 32 \end{bmatrix}$

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2. $\begin{bmatrix} 2 & -3 & 4 \\ 5 & 6 & -7 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix}$

4. $\begin{bmatrix} 3.9 & -2.3 \\ -0.6 & 9.1 \end{bmatrix}$

5. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

6. $\begin{bmatrix} 4 & 3 & 2 \\ 0 & -3 & 5 \end{bmatrix}$

A Practice by Example Example 1 (page 170)

1. **Sports** The modern pentathlon is a grueling all-day competition. Each member of a team competes in five events: target shooting, fencing, swimming, horseback riding, and cross-country running. Find the total scores of the U.S. women's team at the 2000 Olympic Games.

U.S. Women's Pentathlon Scores, 2000 Olympics

Event	Emily deRiel	Mary Beth Jagorashvili
Shoot	1156	964
Fence	800	960
Swim	1182	1205
Ride	1070	1040
Run	1102	960

SOURCE: U.S. Modern Pentathlon Association

- Write two 5×1 matrices to represent the individual scores for each event.
 - Find the total score for the U.S. women's team for each event.
- a-b. See back of book.

Examples 2 and 3 (pages 171 and 172)

Find each sum or difference. 2–9. See margin.

2. $\begin{bmatrix} 2 & -3 & 4 \\ 5 & 6 & -7 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5 \\ 1 & -2 \end{bmatrix}$

4. $\begin{bmatrix} 6.4 & -1.9 \\ -6.4 & 0.8 \end{bmatrix} + \begin{bmatrix} -2.5 & -0.4 \\ 5.8 & 8.3 \end{bmatrix}$

5. $\begin{bmatrix} 6 & -3 \\ -7 & 2 \end{bmatrix} + \begin{bmatrix} -6 & 3 \\ 7 & -2 \end{bmatrix}$

6. $\begin{bmatrix} 5 & 4 & 3 \\ 1 & -2 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

7. $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 2 \\ 3 & 2 & 3 \end{bmatrix}$

8. $\begin{bmatrix} 0.5 & 9.5 \\ -3.5 & 5.5 \end{bmatrix} - \begin{bmatrix} 0.5 & 9.5 \\ -3.5 & 5.5 \end{bmatrix}$

9. $\begin{bmatrix} 1.5 & -1.9 \\ 0 & 4.6 \end{bmatrix} - \begin{bmatrix} 8.3 & -3.2 \\ 2.1 & 5.6 \end{bmatrix}$

Example 4 (page 172)

Solve each matrix equation. 10–13. See margin.

10. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -3 & 4 \end{bmatrix} + X = \begin{bmatrix} 5 & -6 \\ 1 & 0 \\ 8 & 5 \end{bmatrix}$

11. $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix} - X = \begin{bmatrix} 11 & 3 \\ 15 & -9 \end{bmatrix}$

12. $X - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix}$

13. $X + \begin{bmatrix} 6 & 1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$

Example 5 (page 173)

Determine whether the two matrices in each pair are equal. Justify your reasoning.

14. $\begin{bmatrix} -2 & 3 \\ 5 & 0 \end{bmatrix}$, $\begin{bmatrix} 2(-1) & 2(1.5) \\ 2(2.5) & 2(0) \end{bmatrix}$

15. $\begin{bmatrix} 4 \\ -6 \\ -8 \end{bmatrix}$, $[\sqrt{16} \quad -6 \quad \sqrt{64}]$

14–15. See margin pp. 174–175.

Example 6 (page 173)

Find the value of each variable.

16. $\begin{bmatrix} 2 & 2 \\ -1 & 6 \end{bmatrix} - \begin{bmatrix} 4 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} x & y \\ -1 & z \end{bmatrix}$
 $x = -2, y = 3, z = 1$

17. $\begin{bmatrix} 2 & 4 \\ 8 & 12 \end{bmatrix} = \begin{bmatrix} 4x - 6 & -10t + 5x \\ 4x & 15t + 1.5x \end{bmatrix}$
 $x = 2, t = \frac{3}{5}$

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7. $\begin{bmatrix} 0 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix}$

9. $\begin{bmatrix} -6.8 & 1.3 \\ -2.1 & -1 \end{bmatrix}$

11. $\begin{bmatrix} -9 & -2 & 12 \\ -15 & 11 & -7 \end{bmatrix}$

13. $\begin{bmatrix} -4 & -1 \\ -1 & -2 \end{bmatrix}$

8. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

10. $\begin{bmatrix} 4 & -8 \\ -1 & -1 \\ 11 & 1 \end{bmatrix}$

12. $\begin{bmatrix} 6 & 2 \\ -1 & 3 \end{bmatrix}$

14. Yes; $-2 = 2(-1)$, $3 = 2(1.5)$, $5 = 2(2)$ and $0 = 2(0)$ are all correct.

B Apply Your Skills

Use matrices A , B , C , and D . Find each sum or difference if you can. If you cannot, explain why not. **18–22. See back of book.**

$$A = \begin{bmatrix} 3 & 4 \\ 6 & -2 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 1 \\ 2 & -4 \\ -1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix}$$

18. $A + B$ 19. $B + D$ 20. $C + D$ 21. $B - A$ 22. $C - D$

23. **Riding** Use the information in the table below.

U.S. Men's Pentathlon Scores, 2000 World Championship

Event	James Gregory	Velizar Iliev	Chad Senior
Shooting	1132	1072	1072
Fencing	760	910	610
Swimming	1173	1177	1285
Riding	1100	1100	1070
Running	1114	1118	1174

SOURCE: U.S. Modern Pentathlon Association

a–c. See back of book.

- Put the information into three matrices. Label each matrix.
- Find the total score for the U.S. men's pentathlon team for each event.
- Open-Ended** Find the differences between the scores of two of the athletes. In which event were the scores the most different? How different were they?

Solve each matrix equation for X .

24. $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & 3 \end{bmatrix} + X = \begin{bmatrix} 5 & 1 & 8 \\ -6 & 0 & 5 \end{bmatrix}$

25. $X - \begin{bmatrix} 4 & 12 \\ 75 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 50 \\ 50 & -10 \end{bmatrix}$

26. **Data Analysis** Refer to the table.

- Find the total number of people participating in each activity.
- Find the difference between the numbers of males and females participating in each activity.
- Reasoning** In part (b), does the order of the matrices matter? Explain. **a–c. See margin.**

U.S. Participation in Selected Leisure Activities (millions)

Activity	Male	Female
Movies	62.2	65.9
Exercise Programs	70.7	78.1
Sports Events	46.2	34.5
Home Improvement	66.9	61.9

SOURCE: U.S. National Endowment for the Arts. Go to www.PHSchool.com for a data update.

27. **Manufacturing** The table below shows the number of beach balls produced during one shift at two manufacturing plants. Plant 1 has two shifts per day and Plant 2 has three shifts per day.

Beach Ball Production Per Shift

	1-color		3-color	
	Plastic	Rubber	Plastic	Rubber
Plant 1	500	700	1300	1900
Plant 2	400	1200	600	1600

a–b. See back of book.

- Write matrices to represent one day's total output at the two plants.
- Use your results from part (a). Find the difference between production totals at the plants. Which plant produces more three-color plastic balls? Which plant produces more one-color rubber balls?

26a. $\begin{bmatrix} 128.1 \\ 148.8 \\ 80.7 \\ 128.8 \end{bmatrix}$

b. $\begin{bmatrix} -3.7 \\ -7.4 \\ 11.7 \\ 5 \end{bmatrix}$

- c. **Yes, order matters because subtraction is not commutative.**

Lesson Quiz 4-2

Find each sum or difference.

1. $\begin{bmatrix} 2 & 8 \\ 0 & -12 \end{bmatrix} + \begin{bmatrix} -9 & 6 \\ 6 & -5 \end{bmatrix}$

$\begin{bmatrix} -7 & 14 \\ 6 & -17 \end{bmatrix}$

2. $\begin{bmatrix} -3 & 1 \\ 4 & 8 \\ -5 & 4 \end{bmatrix} - \begin{bmatrix} -3 & -2 \\ 9 & 5 \\ 4 & -6 \end{bmatrix}$

$\begin{bmatrix} 0 & 3 \\ -5 & 3 \\ -9 & 10 \end{bmatrix}$

3. What is the additive identity for 2×4 matrix?

$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

4. Solve the equation for x and y .

$\begin{bmatrix} -2x & -1 \\ 5 & x + y \end{bmatrix} = \begin{bmatrix} 18 & -3x + 4y \\ x - 2y & -16 \end{bmatrix}$

$x = -9, y = -7$

5. Are the following matrices equal?

$\begin{bmatrix} 3 & 0.5 \\ 2 & -\frac{2}{3} \end{bmatrix}; \begin{bmatrix} \frac{6}{2} & 0.50 \\ 0.4 & -0.6 \end{bmatrix}$

6. Solve $X - \begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ for the matrix X .

$\begin{bmatrix} 6 & 10 \\ 1 & 11 \end{bmatrix}$

Alternative Assessment

Have students work in pairs. One student writes a 3×2 matrix. Next, each student subtracts the matrix that he or she wrote from the matrix that the partner wrote. They add their result to the sum. The sum should be the 3×2 additive identity matrix. If it is not, each student needs to check the work to discover where the error occurred.

Notice: the matrices have different dimensions.

Resources

For additional practice with a variety of test item formats:

- FCAT Practice, p. 229
- Strategies, p. 224
- FCAT Daily Practice and Strategies Transparencies

Exercises 37, 38 Encourage students to write a matrix for the sum on the left side of the equation. Then solve for any variables.

pages 174–176 Exercises

28a. To find $A + B$ you would add the corresponding elements. To find $A - B$ you would subtract the elements in B from the corresponding elements in A .

b. Matrix C would be the same dimension as A . Its elements would be the opposites of the corresponding elements in A .

29. $a = 2, b = \frac{9}{4}, c = -1, d = 0, f = \frac{1}{2}, g = -4$

30. $x = \pm 3, y = 0$ or 5

31. $c = \frac{5}{2}, d = \frac{2}{5}, f = 7, g = 5, h = -1$

33. Consider any two 2×2 matrices, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

and $B = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$. By the

def. of matrix addition and the Comm. Prop. of Add.,

$$A + B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} w & x \\ y & z \end{bmatrix} =$$

$$\begin{bmatrix} a + w & b + x \\ c + y & d + z \end{bmatrix} =$$

$$\begin{bmatrix} w + a & x + b \\ y + c & z + d \end{bmatrix} = B + A$$

34. Consider any three 2×2 matrices

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} e & f \\ g & h \end{bmatrix},$$

and $C = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$.

By the definition of matrix addition and the Assoc. Prop. of Add.,

$$A + (B + C) =$$

$$\begin{aligned} & \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \left(\begin{bmatrix} e & f \\ g & h \end{bmatrix} + \begin{bmatrix} w & x \\ y & z \end{bmatrix} \right) \\ &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e + w & f + x \\ g + y & h + z \end{bmatrix} \\ &= \begin{bmatrix} a + (e + w) & b + (f + x) \\ c + (g + y) & d + (h + z) \end{bmatrix} \\ &= \begin{bmatrix} (a + e) + w & (b + f) + x \\ (c + g) + y & (d + h) + z \end{bmatrix} \\ &= \begin{bmatrix} a + e & b + f \\ c + g & d + h \end{bmatrix} + \begin{bmatrix} w & x \\ y & z \end{bmatrix} \\ &= (A + B) + C \end{aligned}$$

37. [2] $x = 4, y = -2, w = 0, z = 6$
[1] contains a matrix error

38. [2] $x = 5, y = 1$
[1] contains a matrix error



28. **Writing** Suppose A and B are two matrices with the same dimensions.
- Explain how to find $A + B$ and $A - B$.
 - Explain how to find a matrix C such that $A + C = 0$.

a-b. See margin.

Solve each equation for each variable. 29–31. See margin.

29. $\begin{bmatrix} 4b + 2 & -3 & 4d \\ -4a & 2 & 3 \\ 2f - 1 & -14 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 2c - 1 & 0 \\ -8 & 2 & 3 \\ 0 & 3g - 2 & 1 \end{bmatrix}$

30. $\begin{bmatrix} x^2 & 4 \\ -2 & y^2 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ -2 & 5 \end{bmatrix}$

31. $\begin{bmatrix} 4c & 2 - d & 5 \\ -3 & -1 & 2 \\ 0 & -10 & 15 \end{bmatrix} = \begin{bmatrix} 2c + 5 & 4d & g \\ -3 & h & f - g \\ 0 & -4c & 15 \end{bmatrix}$

Challenge

32. Find the sum of $E = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}$ and the additive inverse of $G = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$.

33. Prove that matrix addition is commutative for 2×2 matrices. See margin.

34. Prove that matrix addition is associative for 2×2 matrices. See margin.

FCAT Practice

Multiple Choice

Use matrices $A = \begin{bmatrix} 5 & 7 & 3 \\ -1 & 0 & -4 \end{bmatrix}$ and $C = \begin{bmatrix} -7 & 4 & 2 \\ 1 & -2 & -3 \end{bmatrix}$ for Exercises 35 and 36.

35. What is the sum $A + C$? **B**

A. The matrices cannot be added.

B. $\begin{bmatrix} -2 & 11 & 5 \\ 0 & -2 & -7 \end{bmatrix}$

C. $\begin{bmatrix} 12 & 3 & 1 \\ -2 & 2 & -1 \end{bmatrix}$

D. $\begin{bmatrix} -35 & 28 & 5 \\ -1 & 0 & 12 \end{bmatrix}$

36. What is matrix Y if $Y - A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$? **G**

F. $\begin{bmatrix} 4 & 7 & 2 \\ -1 & -1 & -5 \end{bmatrix}$

G. $\begin{bmatrix} 6 & 7 & 4 \\ -1 & 1 & -4 \end{bmatrix}$

H. $\begin{bmatrix} -6 & 4 & 3 \\ 1 & -1 & -3 \end{bmatrix}$

I. $\begin{bmatrix} -4 & -7 \\ 1 & 1 \end{bmatrix}$

Find the value of each variable. 37–38. See margin.

37. $\begin{bmatrix} x & y - 2 \\ z & w + 4 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 1 \\ 4 & 8 \end{bmatrix}$

38. $\begin{bmatrix} x & 3 \\ x & -2 \end{bmatrix} + \begin{bmatrix} y & 6 \\ -y & 3 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 4 & 1 \end{bmatrix}$



FCAT Online

FCAT Format quiz at www.PHSchool.com
Web Code: aga-0402

Short Response

Mixed Review

Lesson 4-1

39. **Open-Ended** Write a real-world problem that you can represent with a matrix. Write a matrix for the problem. Label the rows and columns.

Check students' work.

Lesson 3-2

40. **Business** Your friend's mother plans to open a restaurant. The initial investment is \$90,000. Weekly expenses will be about \$8200. If the weekly income is about \$8900, in how many weeks will she get back her investment?

about 129 weeks

Lesson 2-2

Find the slope and y-intercept of the graph of each function.

41. $y = 2x - 6$
2, -6

42. $3y = 6 + 2x$
 $\frac{2}{3}, 2$

43. $-x - 2y = 12$
 $-\frac{1}{2}, -6$

44. $y = 5x$
5, 0