



## Error Prevention

**Exercises 22–30** Students are sometimes intimidated by the stacks of fractions. You may want to suggest that they change the complex fraction to a division problem of the form  $(N) \div (D)$ . Where  $N$  and  $D$  are the expressions in the numerator and denominator of the complex fraction. Be sure to tell them to put  $N$  and  $D$  in tall parentheses. They can then simplify within the tall parentheses, divide, and simplify.

## English Learners

**Exercise 54** You may need to explain the terms *resistance*, *parallel circuit*, and *ohms* to students who are not familiar with the terms.

### Example 4 (page 506)

Simplify each difference.

$$16. \frac{-2}{x} - \frac{1}{x} = -\frac{3}{x}$$

$$18. \frac{y}{2y+4} - \frac{3}{y+2} = \frac{y-6}{2(y+2)}$$

$$20. \frac{3y}{y^2-25} - \frac{8}{y-5} = \frac{-5(y+8)}{(y-5)(y+5)}$$

$$17. \frac{-5y}{2y-1} - \frac{y+3}{2y-1} = \frac{-3(2y+1)}{2y-1}$$

$$19. \frac{x}{3x+9} - \frac{8}{x^2+3x} = \frac{x^2-24}{3x(x+3)}$$

$$21. \frac{2x}{x^2-x-2} - \frac{4x}{x^2-3x+2} = \frac{-2x(x+3)}{(x-2)(x-1)(x+1)}$$

### Example 5 (page 506)

Simplify each complex fraction.

$$22. \frac{\frac{1}{2}}{\frac{x}{2} + \frac{y}{2}}$$

$$23. \frac{1 - \frac{1}{4}}{2 - \frac{3}{5}} = \frac{15}{28}$$

$$24. \frac{\frac{2}{x+y}}{3} = \frac{2}{3(x+y)}$$

$$25. \frac{\frac{1}{3}}{\frac{b}{9}}$$

$$26. \frac{1}{1 + \frac{x}{y}} = \frac{y}{x+y}$$

$$27. \frac{3}{\frac{2}{x} + y} = \frac{3x}{2+xy}$$

$$28. \frac{\frac{2}{x+y}}{\frac{5}{x+y}} = \frac{2}{5}$$

$$29. \frac{\frac{3}{x-4}}{1 - \frac{2}{x-4}} = \frac{3}{x-6}$$

$$30. \frac{-3}{\frac{5}{x} + y} = \frac{-3x}{5+xy}$$

## B Apply Your Skills

Add or subtract. Simplify where possible.

$$31. \frac{3}{4x} - \frac{2}{x^2} = \frac{3x-8}{4x^2}$$

$$32. \frac{3}{x+1} + \frac{x}{x-1} = \frac{x^2+4x-3}{(x+1)(x-1)}$$

$$33. \frac{2x}{x^2-1} - \frac{1}{x^2} = \frac{2x^3-x^2+1}{x^2(x+1)(x-1)}$$

$$34. \frac{4}{x^2-9} + \frac{7}{x+3} = \frac{7x-17}{(x-3)(x+3)}$$

$$35. \frac{x+2}{x-1} - \frac{x-3}{2x+1} = \frac{x^2+9x-1}{(x-1)(2x+1)}$$

$$36. \frac{x}{2x^2-x} + \frac{1}{2x} = \frac{4x-1}{2x(2x-1)}$$

$$37. \frac{5x}{x^2-x-6} - \frac{4}{x^2+4x+4}$$

$$38. 3x + \frac{x^2+5x}{x^2-2} = \frac{x(3x^2+x-1)}{x^2-2}$$

$$37. \frac{5x^2+6x+12}{(x-3)(x+2)^2}$$

$$39. 4y - \frac{y+2}{y^2+3y} = \frac{4y^3+12y^2-y-2}{y(y+3)}$$

$$40. \frac{5y}{y^2-7y} - \frac{4}{2y-14} + \frac{9}{y} = \frac{3(4y)}{y(y)}$$

41. **Open-Ended** Write two rational expressions whose sum is  $\frac{x-2}{x+4}$ .

**Check students' work.**

42. **Factoring is used to determine the least common multiple of the denominators; check students' work.**



42. **Writing** Explain how factoring is used when adding or subtracting rational expressions. Include an example in your explanation.

43. **Error Analysis** How would you convince a student that  $\frac{7x+25}{x^2-9}$  is not the sum of  $\frac{4}{x^2-9}$  and  $\frac{7}{x+3}$ ?

Simplify each complex fraction.

$$44. \frac{\frac{2}{x} + \frac{3}{y}}{\frac{-5}{x} + \frac{7}{y}} = \frac{3x+2y}{7x-5y}$$

$$45. \frac{\frac{5}{x} - \frac{2}{y}}{\frac{-4}{x} - \frac{6}{y}} = \frac{2x-5y}{2(3x+2y)}$$

$$46. \frac{1 + \frac{2}{x}}{2 + \frac{3}{2x}} = \frac{2(x+2)}{4x+3}$$

$$47. \frac{\frac{1}{xy} - \frac{1}{y^2}}{\frac{1}{x^2y} - \frac{1}{xy^2}} = x$$

$$48. \frac{\frac{2}{x+4} + 2}{1 + \frac{3}{x+4}} = \frac{2(x+5)}{x+7}$$

$$49. \frac{\frac{3}{x-2} - 5}{2 - \frac{4}{x-2}} = \frac{3-5(x-2)}{2(x-2)-4}$$

50. **Open-Ended** Write two different complex fractions that simplify to  $\frac{x}{x+4}$ .

**Check students' work.**

51. **Critical Thinking** What real numbers are not in the domain of function  $f$ ? Explain.

$$f(x) = \frac{x+1}{\frac{x+2}{x+3}} \quad x \neq -2, -3, -4; \text{ those values result in division by 0, which is undefined.}$$

# 4. Assess

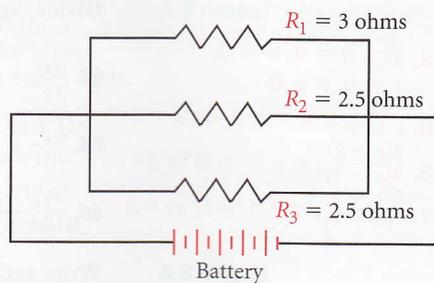


**52. Music** Another kind of average is the harmonic mean. The harmonic mean of two numbers  $a$  and  $b$  equals  $\frac{2}{\frac{1}{a} + \frac{1}{b}}$ . As you vary the length of a violin or guitar string, its pitch changes. If a full-length string is 1 unit long, then many lengths that are simple fractions produce pitches that harmonize, or sound pleasing together. The harmonic mean relates three lengths that produce harmonious sounds. For example,  $\frac{1}{3}$  is the harmonic mean of  $\frac{1}{2}$  and  $\frac{1}{4}$ , and strings of these lengths produce harmonious sounds. Find the harmonic mean for strings of lengths 1 and  $\frac{1}{2}$ ,  $\frac{3}{4}$  and  $\frac{1}{2}$ , and  $\frac{3}{4}$  and  $\frac{2}{3}$ .  **$\frac{2}{3}, \frac{3}{5}, \frac{2}{3}$**

- 53. a.** If you jog 12 mi at an average rate of 4 mi/h and walk the same route back at an average rate of 3 mi/h, you have traveled 24 mi in 7 h and your overall rate is  $\frac{24}{7}$  mi/h. What is your overall average rate if you travel  $d$  mi at 3 mi/h and  $d$  mi at 4 mi/h?  **$\frac{24}{7}$  mi/h**
- b. Harmonic Mean** Find the harmonic mean (see Exercise 52) of 3 and 4.  **$\frac{24}{7}$**
- c.** If you travel any distance at  $x$  mi/h and the same distance at  $y$  mi/h then your average rate for the trip is the harmonic mean of  $x$  and  $y$ . Find the average rate of speed if you travel to a distant city at 50 mi/h and return at 40 mi/h.
- d.** You travel to a city at  $x$  mi/h and the return trip is  $a$  mi/h faster. Express your average rate in terms of  $x$  and  $a$ .

**World Connection**  
Changing a guitar string to  $\frac{2}{3}$  its original length produces a sound in harmony with the original pitch.

**Challenge** **54. Electricity** The total resistance  $R$  for a parallel circuit with three bulbs is  $R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$ .



- a.** Simplify the right side of the equation.
- b.** Find the total resistance  $R$  of a parallel circuit with bulbs that have resistances of 3 ohms, 2.5 ohms, and 2.5 ohms. **0.88 ohms**

**55. Physics** Use the camera lens equation  $\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$ , where  $d_i$  is the distance from the lens to the film and  $d_o$  is the distance from the lens to an object.

**a.** Solve the lens equation for  $f$  by taking the reciprocal of each side of the equation. Simplify the equation so it contains no complex fraction.

**b.** When an object is in focus, a lens is  $x$  cm from the object and  $(2x + 1)$  cm from the film. Find the focal length of the lens.

**a.**  $f = \frac{d_i d_o}{d_i + d_o}$       **b.**  $\frac{x(2x + 1)}{3x + 1} = f$

## FCAT Practice

### Multiple Choice

- 56.** Find the least common multiple of  $x^2 - 1$  and  $x^2 - x$ . **B**
- A.  $x - 1$   
 B.  $x(x - 1)(x + 1)$   
 C.  $x(x - 1)^2(x + 1)$   
 D.  $(x - 1)^2(x + 1)^2x^2$
- 57.** Which expression equals  $\frac{1}{x^2 - 2x - 3} + \frac{1}{x^2 - 4x + 3}$ ? **H**
- F.  $\frac{2x - 1}{(x - 1)(x + 3)(x + 1)}$       G.  $\frac{2x + 1}{(x - 1)(x + 1)(x - 3)}$   
 H.  $\frac{2x}{(x - 1)(x + 1)(x - 3)}$       I.  $\frac{2x}{(x + 3)(x - 1)(x + 1)}$

## Lesson Quiz 9-5

- 1.** The lens equation for a camera is  $\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$ , where  $f$  is the focal length of the lens,  $d_i$  is the distance between the lens and the film, and  $d_o$  is the distance between the lens and the object. For a certain camera, an object that is 18 cm from the lens is in focus when the lens is 9 cm from the film. What is the focal length of the lens? **6 cm**
- 2.** Find the least common multiple of  $x(x^2 - 5x + 6)$  and  $x^3(x^2 + 4x - 21)$ .  
 **$x^3(x - 2)(x - 3)(x + 7)$**

Simplify.

- 3.**  $\frac{x}{x^2 - 4} + \frac{7}{3x + 6}$        **$\frac{10x - 14}{3x^2 - 12}$**
- 4.**  $\frac{m}{m + 3} - \frac{6m}{m^2 - 9}$        **$\frac{m^2 - 9m}{m^2 - 9}$**
- 5.**  $\frac{2y}{2y + 1} - \frac{1}{1 - \frac{2y}{2y - 1}}$        **$\frac{2y - 1}{2y + 1}$**

## Alternative Assessment

Have students work individually. Each student writes two rational expressions with binomial numerators. One rational expression should have  $x^2 - 4$  as its denominator, and the other should have  $x + 2$  as its denominator. They find the sum and difference of their expressions. Then have each student write a complex fraction that has the original expressions as numerator and denominator. The student then simplifies the complex fraction.

## FCAT Practice

### Resources

- For additional practice with a variety of test item formats:
- FCAT Practice, p. 531
  - FCAT Strategies, p. 526
  - FCAT Daily Practice and Strategies Transparencies

# Chapter Checkpoint 2

To check understanding of Lessons 9-4 to 9-5:

Checkpoint Quiz 2 (p. 510)

**Teaching Resources**  
Checkpoint Quiz 2 (also in Prentice Hall Assessment System)

**Reaching All Students**  
Reading and Math Literacy 9C

Spanish versions available

## Exercises 507-510

**60. [2] First factor both denominators:**  
 $x^2 - 5x - 6 = (x - 6)(x + 1)$   
 $x^2 - 12x + 36 = (x - 6)^2$ . The least common denominator would have to include the factors  $(x - 6)$ ,  $(x + 1)$  and  $(x - 6)^2$ , so the LCD is  $(x - 6)^2(x + 1) = x^3 - 11x^2 + 24x + 36$ .

[1] answer only, no work shown

**61. [4]**  $\frac{2 - \frac{15}{x}}{\frac{3}{x}} = 9$

$$\frac{2x - 15}{\frac{3}{x}} = 9$$

Find a common denominator for the numerator.

$$\frac{2x - 15}{\frac{x}{x}} = 9$$

Combine the numerators.

$$\frac{2x - 15}{x} \cdot \frac{x}{3} = 9$$

Multiply the numerator by the reciprocal of the denominator.

$$\frac{2x - 15}{3} = 9$$

Solve for  $x$ .

$$2x - 15 = 27$$

$$2x = 42$$

$$x = 21$$

[3] one calculation error



**FCAT Online**

FCAT Format quiz at [www.PHSchool.com](http://www.PHSchool.com)  
Web Code: aga-0905

### Short Response

**58.** Which expression equals  $\frac{5x}{x^2 - 9} - \frac{4x}{x^2 + 5x + 6}$ ? **C**

A.  $\frac{7x}{(x - 3)(x + 3)(x + 2)}$

B.  $\frac{x^2 - 2x}{(x - 3)(x + 3)(x + 2)}$

C.  $\frac{x^2 + 22x}{(x - 3)(x + 3)(x + 2)}$

D.  $\frac{9x^2 - 2x}{(x - 3)(x + 3)(x + 2)}$

**59.** Simplify  $\frac{\frac{2}{x} - 5}{\frac{6}{x} - 3}$ . **F**

F.  $\frac{2 - 5x}{6 - 3x}$

G.  $\frac{2 + 5x}{6 - 3x}$

H.  $\frac{2x - 5}{6x + 3}$

I.  $\frac{6 + 3x}{2}$

**60.** Find the least common denominator for the rational expressions and  $\frac{1}{x^2 - 12x + 36}$ . Show your work. **See margin.**

**61.** Solve the equation  $\frac{2 - \frac{15}{x}}{\frac{3}{x}} = 9$ . Show your work. **See margin.**

## Mixed Review

### Lesson 9-4

**63.**  $\frac{1}{4xy}$ ;  $a \neq 0, b \neq 0, x \neq 0, y \neq 0$

**65.**  $\frac{12x}{x + 3}$ ;  $x \neq -3, 2, \text{ or } 3$

**66.**  $\frac{3(x - 2)}{4(x - 3)}$ ;  $x \neq -2, 2, \text{ or } 3$

**67.**  $\frac{3(x + 1)}{2(x + 3)}$ ;  $x \neq 1, -1, \text{ or } -3$

**Divide. State any restrictions on the variables.**

**62.**  $\frac{4x^3}{3y^4} \div \frac{16x^2}{9y^2}$   $\frac{3x}{4y^2}$ ;  $x \neq 0, y \neq 0$

**63.**  $\frac{7ax^3}{8by^2} \div \frac{14ax^4}{4by}$

**64.**  $\frac{6x^2}{y} \div \frac{12x^4}{y^3}$   $\frac{y^2}{2x^2}$ ;  $x \neq 0, y \neq 0$

**65.**  $\frac{3x^2 - 9x}{x - 2} \div \frac{x^2 - 9}{4x - 8}$

**66.**  $\frac{3x - 6}{12x + 24} \div \frac{x^2 - 5x + 6}{3x^2 - 12}$

**67.**  $\frac{5x + 15}{10x - 10} \div \frac{x^2 + 6x + 9}{3x^2 - 3}$

### Lesson 8-4

**Write each expression as a single logarithm.**

**68.**  $\log_3 y + 4 \log_3 t$   
 $\log_3 t^4 y$

**69.**  $7 \log_{10} p + \log_{10} q$   
 $\log_{10} p^7 q$

**70.**  $\log_5 x - \frac{1}{5} \log_5 \frac{x}{y}$

### Lesson 7-6

**Let  $f(x) = x^2 + 1$  and  $g(x) = 3x$ . Find each value.**

**71.**  $(g \circ f)(-3)$   
**30**

**72.**  $(f \circ g)(-3)$   
**82**

**73.**  $(g \circ f)\left(\frac{1}{2}\right)$   
 **$\frac{15}{4}$**

**74.**  $(f \circ f)$   
**101**



## Checkpoint Quiz 2

Lessons 9-4 through 9-5



Instant self-check quiz online and on CD-ROM

**Simplify each expression.**

**1.**  $\frac{3x - 6}{5x - 20} \cdot \frac{x - 8}{5x - 10}$   $\frac{3(x - 8)}{25(x - 4)}$

**2.**  $\frac{14x + 7}{4x - 6} \cdot \frac{8x - 12}{42x + 21}$   $\frac{2}{3}$

**3.**  $\frac{y^2 - 25}{(y + 5)^2} \div \frac{2y - 10}{4y + 20}$  **2**

**4.**  $\frac{y^2 - 25}{y^2 - 16} \div \frac{2y + 10}{y^2 - 4y}$   $\frac{y(y - 5)}{2(y + 4)}$

**5.**  $\frac{8}{3x^3y} + \frac{4}{9xy^3}$   $\frac{4(6y^2 + x^2)}{9x^3y^3}$

**6.**  $\frac{7}{5y + 25} + \frac{4}{3y + 15}$   $\frac{41}{15(y + 5)}$

**7.**  $\frac{5x}{2y + 4} - \frac{6}{y^2 + 2y}$   $\frac{5xy - 12}{2y(y + 2)}$

**8.**  $3x - \frac{x^2 + 5x}{x^2 - 2}$   $\frac{3x^3 - x^2 - 1}{x^2 - 2}$

**9.**  $\frac{\frac{3}{2y}}{\frac{6}{8x}}$   $\frac{2x}{y}$

**10.**  $\frac{\frac{1}{x} + 3}{4 + \frac{5}{y}}$   $\frac{y(3x + 1)}{x(4y + 5)}$

## 510 Chapter 9 Rational Functions

**[2] one process error, such as an error in simplifying the numerator**

**[1] answer only, no work shown**