. Practice

Assignment Guide

1 Objective

A B Core 1–80

C Extension 81–87

Standardized Test Prep 88–94

Mixed Review 95–106

r Prevention

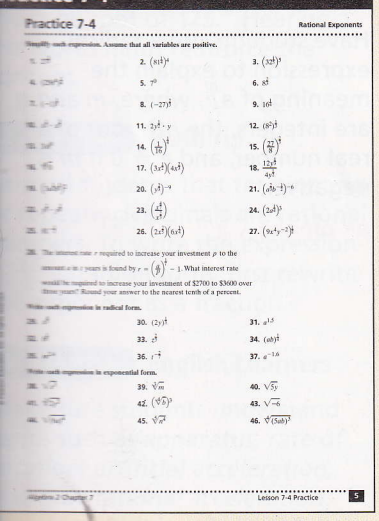
rcises 18–21 Be sure students ecall that the index for a square oot expression is 2.

xercises 65–76 You may need o remind students that the xpressions in each final answer hould contain only positive xponents.

Enrichment 7-4

eteaching 7-4

Practice 7-4



To write an expression with rational exponents in simplest form, write every exponent as a positive number.

5 EXAMPLE Writing Expressions in Simplest Form

Write $(16y^{-8})^{-\frac{3}{4}}$ in simplest form.

$$(16y^{-8})^{-\frac{3}{4}} = (2^4y^{-8})^{-\frac{3}{4}}$$
$$= 2^{4\cdot(-\frac{3}{4})} \cdot y^{-8\cdot(-\frac{3}{4})}$$
$$= 2^{-3}y^6$$
$$= \frac{y^6}{2^3}$$
$$= \frac{y^6}{8}$$

Check Understanding 5 Write $(8x^{15})^{-\frac{1}{3}}$ in simplest form. $\frac{1}{2x^5}$

EXERCISES

For more practice, see *Extra Prac*

Practice and Problem Solving

A Practice by Example

Example 1 (page 379)

Simplify each expression.

1. $36^{\frac{1}{2}}$ 6
2. $27^{\frac{1}{3}}$ 3
3. $49^{\frac{1}{2}}$ 7
4. $10^{\frac{1}{2}} \cdot 10^{\frac{1}{2}}$ 10
5. $(-3)^{\frac{1}{3}} \cdot (-3)^{\frac{1}{3}} \cdot (-3)^{\frac{1}{3}}$ −3
6. $3^{\frac{1}{2}} \cdot 12^{\frac{1}{2}}$ 6
7. $2^{\frac{1}{2}} \cdot 32^{\frac{1}{2}}$ 8
8. $3^{\frac{1}{3}} \cdot 9^{\frac{1}{3}}$ 3
9. $3^{\frac{1}{4}} \cdot 27^{\frac{1}{4}}$ 3

Example 2 (page 380)

Write each expression in radical form.

10. $x^{\frac{1}{6}}$ $\sqrt[6]{x}$
11. $x^{\frac{1}{5}}$ $\sqrt[5]{x}$
12. $x^{\frac{2}{7}}$ $\sqrt[7]{x^2}$ or $(\sqrt[7]{x})^2$
13. $y^{\frac{2}{5}}$ $\sqrt[5]{y^2}$ or $(\sqrt[5]{y})^2$
14. $y^{-\frac{9}{8}}$ $\frac{1}{\sqrt[8]{y^9}}$ or $\frac{1}{(\sqrt[8]{y})^9}$
15. $t^{-\frac{3}{4}}$ $\frac{1}{\sqrt[4]{t^3}}$ or $\frac{1}{(\sqrt[4]{t})^3}$
16. $x^{1.5}$ $\sqrt{x^3}$ or $(\sqrt{x})^3$
17. $y^{1.2}$ $\sqrt[5]{y^6}$ or $(\sqrt[5]{y})^6$

Write each expression in exponential form.

18. $\sqrt{-10}$ $(-10)^{\frac{1}{2}}$
19. $\sqrt{7x^3}$ $7^{\frac{1}{2}}x^{\frac{3}{2}}$
20. $\sqrt{(7x)^3}$ $(7x)^{\frac{3}{2}}$
21. $(\sqrt{7x})^3$ $(7x)^{\frac{3}{2}}$
22. $\sqrt[3]{a^2}$ $a^{\frac{2}{3}}$
23. $(\sqrt[3]{a})^2$ $a^{\frac{2}{3}}$
24. $\sqrt[4]{c^2}$ $c^{\frac{1}{2}}$
25. $\sqrt[3]{(5xy)^2}$ $25x^2y^2$

Example 3 (page 380)

The optimal height h of the letters of a message printed on pavement is given by the formula $h = \frac{0.00252d^{2.27}}{e}$. Here d is the distance of the driver from the letters and e is the height of the driver's eye above the pavement. All of the distances are in meters. Find h for the given values of d and e.

26. $d = 100$ m, $e = 1.2$ m ≈72.8 m
27. $d = 50$ m, $e = 1.2$ m ≈15.1 m
28. $d = 50$ m, $e = 2.3$ m ≈7.9 m
29. $d = 25$ m, $e = 2.3$ m ≈1.6 m

Example 4 (page 381)

Simplify each number.

30. $8^{\frac{2}{3}}$ 4
31. $64^{\frac{2}{3}}$ 16
32. $(-8)^{\frac{2}{3}}$ 4
33. $(-32)^{\frac{2}{5}}$
34. $(32)^{-\frac{4}{5}}$ $\frac{1}{16}$
35. $4^{1.5}$ 8
36. $16^{1.5}$ 64
37. $10{,}000^{0.75}$

Example 5 (page 382)

40. $\frac{1}{3x^{\frac{2}{3}}}$

41. $\frac{5}{x^{\frac{2}{3}}}$

45. $x^4\sqrt[3]{x}$

Write each expression in simplest form. Assume that all variables are positive.

38. $(x^{\frac{2}{3}})^{-3}$ $\frac{1}{x^2}$ 39. $(x^{-\frac{4}{7}})^7$ $\frac{1}{x^4}$ 40. $(3x^{\frac{2}{3}})^{-1}$ 41. $5(x^{\frac{2}{3}})^{-1}$

42. $(-27x^{-9})^{\frac{1}{3}}$ $-\frac{3}{x^3}$ 43. $(-32y^{15})^{\frac{1}{5}}$ $-2y^3$ 44. $\left(\frac{x^3}{x^{-1}}\right)^{-\frac{1}{4}}$ $\frac{1}{x}$ 45. $\left(\frac{x^2}{x^{-11}}\right)^{\frac{1}{3}}$

46. $(x^{\frac{1}{2}}y^{-\frac{2}{3}})^{-6}$ $\frac{y^4}{x^3}$ 47. $(x^{\frac{2}{3}}y^{-\frac{1}{6}})^{-12}$ $\frac{y^2}{x^8}$ 48. $\left(\frac{x^{\frac{1}{4}}}{y^{-\frac{3}{4}}}\right)^{12}$ x^3y^9 49. $\left(\frac{x^{-\frac{2}{3}}}{y^{-\frac{1}{3}}}\right)^{15}$ $\frac{y^5}{x^{10}}$

B Apply Your Skills

Simplify each number.

50. $(-343)^{\frac{1}{3}}$ -7 51. $(-243)^{\frac{1}{5}}$ -3 52. $32^{1.2}$ 64 53. $243^{1.2}$ 729

54. $64^{3.5}$ 2,097,152 55. $100^{4.5}$ 1,000,000,000 or 10^9 56. $32^{-0.4}$ $\frac{1}{4}$ 57. $64^{-0.5}$ $\frac{1}{8}$

58. $(-216)^{-\frac{2}{3}}$ $\frac{1}{36}$ 59. $2(16)^{\frac{3}{4}}$ 16 60. $-(-27)^{-\frac{4}{3}}$ $-\frac{1}{81}$ 61. $\frac{1000^{\frac{4}{3}}}{100^{\frac{3}{2}}}$ 10

62. **Archaeology** The ratio R of radioactive carbon to nonradioactive carbon left in a sample of an organism that died T years ago can be approximated by the formula $R = A(2.7)^{-\frac{T}{8033}}$. Here A is the ratio of radioactive carbon to nonradioactive carbon in the living organism. What percent of A is left after 2000 years? After 4000 years? After 8000 years? 78%, 61%, 37%

63. **Biology** The expression $0.036m^{\frac{3}{4}}$ is used in the study of fluids. Evaluate the expression for $m = 46 \times 10^4$. 635.87

64. **Physics** In the expression $PV^{\frac{7}{5}}$, P represents the pressure and V represents the volume of a sample of a gas. Evaluate the expression for $P = 6$ and $V = 32$. 768

World Connection

ologists estimate the artifacts and fossils by xponential functions.

Simplify each expression. Assume that all variables are positive.

65. $x^{\frac{2}{7}} \cdot x^{\frac{3}{14}}$ $x^{\frac{1}{2}}$ 66. $y^{\frac{1}{2}} \cdot y^{\frac{3}{10}}$ $y^{\frac{4}{5}}$ 67. $x^{\frac{3}{5}} \div x^{\frac{1}{10}}$ $x^{\frac{1}{2}}$ 68. $y^{\frac{5}{7}} \div y^{\frac{3}{14}}$ $y^{\frac{1}{2}}$

69. $\frac{x^{\frac{2}{3}}y^{-\frac{1}{4}}}{x^{\frac{1}{2}}y^{-\frac{1}{2}}}$ $x^{\frac{1}{6}}y^{\frac{1}{4}}$ 70. $\frac{x^{\frac{1}{2}}y^{-\frac{1}{3}}}{x^{\frac{3}{4}}y^{\frac{1}{2}}}$ $\frac{1}{x^{\frac{1}{4}}y^{\frac{5}{6}}}$ 71. $\left(\frac{16x^{14}}{81y^{18}}\right)^{\frac{1}{2}}$ $\frac{4x^7}{9y^9}$ 72. $\left(\frac{81y^{16}}{16x^{12}}\right)^{\frac{1}{2}}$ $\frac{9y^8}{4x^6}$

73. $(x^{\frac{1}{2}} \cdot x^{\frac{5}{12}})^{\frac{1}{3}} \div x^{\frac{2}{3}}$ 74. $(x^{\frac{3}{4}} \div x^{\frac{7}{8}}) \cdot x^{-\frac{1}{6}}$ 75. $[(x^{-\frac{1}{2}})^2]^{\frac{1}{3}}$ 76. $[(\sqrt{x^3y^3})^{\frac{1}{3}}]^{-1}$

77. **Writing** Explain why $(-64)^{\frac{1}{3}} = -64^{\frac{1}{3}}$ and $(-64)^{\frac{1}{2}} \neq -64^{\frac{1}{2}}$. See margin.

78. **Error Analysis** Explain why the following simplification is incorrect.
$5(4 - 5^{\frac{1}{2}}) = 5(4) - 5(5^{\frac{1}{2}}) = 20 - 25^{\frac{1}{2}} = 15$ See margin.

79. a. **Open-Ended** Find three numbers a such that $a(4 + 5^{\frac{1}{2}})$ is a rational number.
b. **Critical Thinking** Are there any rational numbers a such that $a(4 + 5^{\frac{1}{2}})$ is a rational number? no

Answers may vary. Sample: $4 - 5^{\frac{1}{2}}$, $2(4 - 5^{\frac{1}{2}})$, $\frac{4 - 5^{\frac{1}{2}}}{2}$

80. a. **Reasoning** Show that $\sqrt[4]{x^2} = \sqrt{x}$ by using the definition of fourth root.
b. **Reasoning** Show that $\sqrt[4]{x^2} = \sqrt{x}$ by rewriting $\sqrt[4]{x^2}$ in exponential form.
a–b. See margin.

C Challenge

Exponents that are irrational numbers can be defined so that all the properties of rational exponents are also true for irrational exponents. Use those properties to simplify each expression.

81. $(7^{\sqrt{2}})^{\sqrt{2}}$ 49 82. $\frac{3^{3+\sqrt{5}}}{3^{1+\sqrt{5}}}$ 9 83. $\frac{x^{4\pi}}{x^{2\pi}}$ $x^{2\pi}$

84. $5^{2\sqrt{3}} \cdot 25^{-\sqrt{3}}$ 1 85. $9^{\frac{1}{\sqrt{2}}}$ $3^{\sqrt{2}}$ 86. $(3^{2+\sqrt{2}})^{2-\sqrt{2}}$ 9

4. Assess

Lesson Quiz 7-4

1. Simplify each expression.
a. $100^{\frac{1}{2}}$ 10 b. $(-64)^{\frac{1}{3}}$ -4
2. Write each expression in radical form.
a. $x^{\frac{1}{7}}$ $\sqrt[7]{x}$
b. $y^{\frac{4}{5}}$ $\sqrt[5]{y^4}$ or $(\sqrt[5]{y})^4$
c. $k^{1.8}$ $\sqrt[5]{k^9}$ or $(\sqrt[5]{k})^9$
d. $a^{-\frac{1}{6}}$ $\frac{1}{\sqrt[6]{a}}$
3. A container with curved sides is 10 ft tall. When water is in the container to a depth of h ft, the number of cubic feet of water can be approximated by using the formula $V = 0.95h^{2.9}$. Find the amount of water in the container when the depth of the water is 7.5 ft. Round to the nearest hundredth. 327.64 ft^3
4. Simplify each number.
a. $(-64)^{\frac{4}{3}}$ 256
b. $81^{0.75}$ 27
5. Write $(x^{\frac{2}{3}}y^{-\frac{1}{4}})^{-4}$ in simplest form. $\frac{y}{x^{\frac{8}{3}}}$

Alternative Assessment

Have students work in pairs. Each student should write a new exercise similar to those in each exercise group in Practice and Problem Solving, Section A (pages 382–383). Students trade exercises and solve the exercises that his or her partner wrote. Students check each other's work and discuss any discrepancies.

382–384 Exercises

e cube root of -64 is 4, which equals $-(64)^{\frac{1}{3}}$. e square root of -64 is t a real number, but $(64)^{\frac{1}{2}} = -\sqrt{64} = -8$.

78. The exponent $\frac{1}{2}$ applies only to the 5, not to the 25.

80a. $\sqrt{x} \cdot \sqrt{x} \cdot \sqrt{x} \cdot \sqrt{x} = x \cdot x = x^2$, so $\sqrt[4]{x^2} = \sqrt{x}$
b. $\sqrt[4]{x^2} = (x^2)^{\frac{1}{4}} = x^{\frac{2}{4}} = x^{\frac{1}{2}} = \sqrt{x}$

FCAT Practice

Resources
For additional practice with a variety of test item formats:
- FCAT Practice, p. 419
- Strategies, p. 414
- FCAT Daily Practice and Strategies Transparencies

Exercises 88, 89 It may help to write each expression in simplest form with fractional exponents.

pages 382–384 Exercises

98. $15 - 4\sqrt{14}$
99. $\frac{4 + 6\sqrt{5} + 2\sqrt{10} + 15\sqrt{2}}{-41}$
100. $\frac{10 - 8\sqrt{2}}{7}$
101. $4x(x^2 - 2x + 4)$
102. $(x + 2)^2$
103. $(x - 9)^2$
104. $(4a - 3b)(4a + 3b)$
105. $(5x - 4y)^2$
106. $(3x + 8)^2$

87. **Weather** Using data for the effect of temperature and wind on an expos face, the National Weather Service uses the following formula.

$$\text{Wind Chill Index} = 35.74 + 0.6215T - 35.75V^{0.16} + 0.4275TV^{0.16}$$

T is the temperature in degrees Fahrenheit and V is the velocity of the w miles per hour. Frostbite occurs in about 15 minutes when the wind chill is about -20. Find the wind speed that produces a wind chill index of -2 when the temperature is 5°F. **33.13 mph**

FCAT Practice

Multiple Choice

88. Which expression is NOT equivalent to $\sqrt[4]{4n^2}$? **B**
A. $(4n^2)^{\frac{1}{4}}$ B. $2n^{\frac{1}{2}}$ C. $(2n)^{\frac{1}{2}}$ D. $\sqrt{2n}$

89. Which expression is NOT equivalent to $\sqrt[6]{81x^4y^8}$? **H**
F. $(3xy^2)^{\frac{2}{3}}$ G. $(3x)^{\frac{2}{3}}y^{\frac{4}{3}}$ H. $(3x^2y^4)^{\frac{1}{3}}$ I. $\sqrt[3]{9x}$

Short Response

90. What is the value of x if $32^{0.8}x = 1$? Simplify the answer. **See left.**

Multiple Choice

Compare the boxed quantity in Column A with the boxed quantity in Column B. Choose the best answer.
A. The quantity in Column A is greater.
B. The quantity in Column B is greater.
C. The two quantities are equal.
D. The relationship cannot be determined from the information give

90. **[2]** $x = \frac{1}{16}$
[1] minor error

FCAT Online
FCAT Format quiz at www.PHSchool.com
Web Code: aga-0704

	Column A	Column B
91. B	$14^{\frac{2}{3}}$	$14^{\frac{3}{2}}$
92. A	$5^{-\frac{3}{4}}$	$5^{-\frac{4}{3}}$
93. C	$16^{-\frac{3}{4}}$	$32^{-\frac{3}{5}}$
94. A	$(-17)^{100}$	$(-17)^{101}$

Mixed Review

Lesson 7-3 **Simplify. Rationalize all denominators.**

95. $6\sqrt[3]{3} - 2\sqrt[3]{3}$ **$4\sqrt[3]{3}$**
96. $3\sqrt{18} + 2\sqrt{72}$ **$21\sqrt{2}$**
97. $(\sqrt{5} - 1)(\sqrt{5}$ **$1 +$**
98. $(\sqrt{8} - \sqrt{7})^2$
99. $\frac{2 + \sqrt{10}}{2 - 3\sqrt{5}}$
100. $\frac{-2 + \sqrt{8}}{-3 - \sqrt{2}}$

Lesson 5-4 **Factor each expression.** **98–106. See margin.**

101. $4x^3 - 8x^2 + 16x$
102. $x^2 + 4x + 4$
103. $x^2 - 18x +$
104. $16a^2 - 9b^2$
105. $25x^2 - 40xy + 16y^2$
106. $9x^2 + 48x +$