Additional Examples

4 Jim and Alberto have to paint 6000 square feet of hallway in an office building. Alberto works twice as fast as Jim. Working together, they can complete the job in 15 hours. How long would it take each of them working alone? **Alberto: 22.5 h, Jim: 45 h**

Closure

Ask students to describe what they must do to find the solutions of a rational equation. **Answers may vary. Sample: First, clear the equation of fractions. One way to do this is to simplify both sides and then multiply both sides of the resulting equation by the least common denominator of all expressions. The result will be a polynomial equation. Solve this equation. Check the solutions in the original equation to eliminate extraneous solutions.**

4 EXAMPLE Real-World Connection

Volunteerism Tim can stuff envelopes three times as fast as his daughter Ge They have to stuff 5000 envelopes for a fund-raiser. Working together, Tim a Georgia can complete the job in four hours. How long would it take each of working alone?

Relate Tim's rate + Georgia's rate = combined rate

Define

	Time (hr)	Rate (envelopes per hour)
Tim	x	$\frac{5000}{x}$
Georgia	$3x$	$\frac{5000}{3x}$
Combined	4	$\frac{5000}{4} = 1250$

Write $\frac{5000}{x} + \frac{5000}{3x} = 1250$

$$\frac{5000}{x} + \frac{5000}{3x} = 1250$$

$$3x\left(\frac{5000}{x} + \frac{5000}{3x}\right) = 3x(1250) \quad \text{Multiply by the LCD, } 3x.$$

$$\frac{3x(5000)}{x} + \frac{3x(5000)}{3x} = 3x(1250) \quad \text{Distributive Property.}$$

$$15{,}000 + 5000 = 3750x \quad \text{Simplify.}$$

$$20{,}000 = 3750x \quad \text{Simplify.}$$

$$5.33 \approx x \quad \text{Solve for } x.$$

Tim could stuff 5000 envelopes in about 5.33 hours.
Georgia could stuff 5000 envelopes in 3(5.33) hours, or about 16 hours.

Check Understanding 4 a. Suppose Maria can stuff envelopes twice as fast as her friend Paco. Tog they can stuff 6750 envelopes in 4.5 hours. How long would it take each working alone? **Maria: 6.75 h, Paco: 13.5 h**

b. Suppose Adrian can weed the garden twice as fast as his son Phillip. To they can weed the garden in 3 hours. How long would it take each of th working alone? **Adrian: 4.5 h, Phillip: 9 h**

EXERCISES

For more practice, see Extra

Practice and Problem Solving

A Practice by Example

Example 1 (page 512)

Solve each equation. Check each solution.

1. $\frac{x}{5} = \frac{x+3}{8}$ **5**
2. $\frac{1}{5x} = \frac{1}{9x}$ **no solution**
3. $\frac{4}{3x+3} =$
4. $\frac{2}{x-1} = \frac{x+4}{3}$ **2 or −5**
5. $\frac{3}{x+1} = \frac{1}{x^2-1}$ **$\frac{4}{3}$**
6. $\frac{4}{2x-3} =$
7. $\frac{3}{x} = \frac{12}{x+7}$ **$\frac{7}{3}$**
8. $\frac{10}{6x+7} = \frac{6}{2x+9}$ **3**
9. $\frac{2}{3x-5} =$

Example 2 (page 513)

Solve each equation. Check each solution.

10. $\frac{1}{4} - x = \frac{x}{8}$ $\frac{2}{9}$

11. $\frac{y}{5} + \frac{y}{2} = 7$ 10

12. $\frac{2x}{3} - \frac{1}{2} = \frac{2x+5}{6}$ 4

13. $\frac{3x-2}{12} - \frac{1}{6} = \frac{1}{6}$ 2

14. $\frac{1}{x} + \frac{x}{2} = \frac{x+4}{2x}$ −1 or 2

15. $\frac{11}{3x} - \frac{1}{3} = \frac{-4}{x^2}$ −1 or 12

16. $\frac{3}{2x} - \frac{5}{3x} = 2$ $-\frac{1}{12}$

17. $\frac{5x}{4} - \frac{3}{x} = \frac{1}{4}$ −1.45 or 1.65

18. $\frac{2}{y} + \frac{1}{2} = \frac{5}{2y}$ 1

19. $x + \frac{6}{x} = -5$ −3, −2

20. $\frac{1}{4x} - \frac{3}{4} = \frac{7}{x}$ −9

21. $\frac{5}{2x} - \frac{2}{3} = \frac{1}{x} + \frac{5}{6}$ 1

Examples 3 and 4 (pages 513 and 514)

22. Carlos can travel 40 mi on his motorbike in the same time it takes Paul to travel 15 mi on his bicycle. If Paul rides his bike 20 mi/h slower than Carlos rides his motorbike, find the speed for each bike. Carlos: 32 mi/h, Paul: 12 mi/h

23. A passenger train travels 392 mi in the same time that it takes a freight train to travel 322 mi. If the passenger train travels 20 mi/h faster than the freight train, find the speed of each train. passenger train: 112 mi/h, freight train: 92 mi/h

24. Shelley can paint a fence in 8 hours. Karen can do it in 4 hours. How long will it take them to do the job if they work together? $2\frac{2}{3}$ h

25. One pump can fill a tank with oil in 4 hours. A second pump can fill the same tank in 3 hours. If both pumps are used at the same time, how long will they take to fill the tank? $1\frac{5}{7}$ h

Apply Your Skills

Solve each equation for the given variable. 27–31. See margin.

26. $m = \frac{2E}{V^2}$; E $E = \frac{mV^2}{2}$

27. $\frac{c}{E} - \frac{1}{mc} = 0$; E

28. $\frac{m}{F} = \frac{1}{a}$; F

29. $\frac{1}{c} - \frac{c}{a^2 - b^2} = 0$; c

30. $\frac{\ell}{T^2} = \frac{g}{4\pi^2}$; T

31. $\frac{q}{m} = \frac{2V}{B^2r^2}$; B

Reading Math

r help with reading d solving Exercise 32, e page 518.

32. Anita and Fran have volunteered to contact every member of their organization by phone to inform them of an upcoming event. Fran can complete the calls in six days if she works alone. Anita can complete them in four days. How long will they take to complete the calls working together? $2\frac{2}{5}$ days

33. **Test Scores** On the first four tests of the term your average is 84%. You think you can score 96% on each of the remaining tests. How many consecutive test scores of 96% would you need to bring your average up to 90% for the term? 4

34. You are planning a school field trip to a local theater. It costs \$60 to rent the bus. Each theater ticket costs \$5.50. $c(x) = \frac{5.50x + 60}{x}$

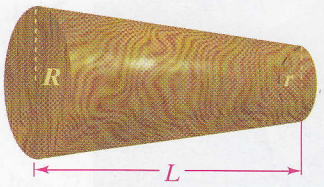
a. Write a function $c(x)$ to represent the cost per student if x students sign up.

b. How many students must sign up if the cost is to be no more than \$10 per student? 14 students

a. $L = \frac{24(R-r)}{T}$

b. 32 in., 28.24 in., 25.26 in.

35. **Woodworking** A tapered cylinder is made by decreasing the radius of a rod continuously as you move from one end to the other. The speed at which it tapers is the taper per foot. You can calculate the taper per foot using the formula $T = \frac{24(R-r)}{L}$. The lengths R, r, and L are measured in inches.



a. Solve this equation for L.

b. Find L if $R = 4$ in.; $r = 3$ in.; and $T = 0.75, 0.85$, and 0.95.

3. Practice

Assignment Guide

1 Objective

A B Core 1–21, 26–31, 39–52

C Extension 56

2 Objective

A B Core 22–25, 32–38, 53–55

C Extension 57, 58

Standardized Test Prep 59–64

Mixed Review 65–74

Error Prevention

Exercises 10–21, 39–52 Students may forget to check for extraneous solutions. Stress that this is an essential step when solving rational equations.

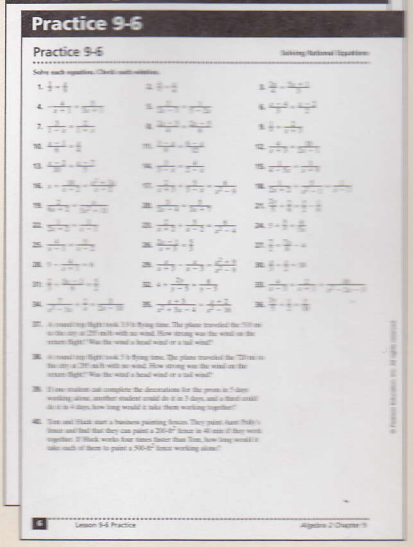
English Learners

Exercise 35 You may need to explain the terms *tapered cylinder* and *continuously*.

Enrichment 9-6

Reteaching 9-6

Practice 9-6



514–517 Exercises

= mc^2

= ma

= $\sqrt{a^2 - b^2}$

30. $T = \sqrt{\frac{4\pi^2\ell}{g}}$

31. $B = \sqrt{\frac{2Vm}{r^2q}}$

4. Assess

Lesson Quiz 9-6

Solve each equation. Check each solution.

1. $\frac{5x}{x^2 - 25} = \frac{2}{x - 5}$ $\frac{10}{3}$
2. $1 + \frac{2}{x} = \frac{3}{x^2}$ $-3, 1$
3. $\frac{x}{x - 1} + x = \frac{4x - 3}{x - 1}$ 3
4. The speed of the current in a river is 5 miles per hour. A boat leaves a dock on the bank of the river, travels upstream 25 miles, and returns to the dock in 12 hours. What is the speed of the boat in still water? 7.5 mi/h

Alternative Assessment

Ask each student to write two real-world problems about motion (problems that involve distance, speed, and time) that can be modeled by a rational equation. Ask them to write, solve, and check an equation to solve each problem. Students can exchange problems to check each other's work.

36. **Fuel Economy** Suppose you drive an average of 15,000 miles per year, an your car gets 24 miles per gallon. Suppose gasoline costs $1.60 a gallon.
   a. How much money do you spend each year on gasoline? $1000
   b. You plan to trade in your car for one that gets x more miles per gallon Write an expression to represent the new yearly cost of gasoline.
   c. Write an expression to represent your savings on gasoline.
   d. Suppose you save $200 a year with the new car. How many miles per does the new car get? 30 mi/gal

36b. $\frac{15,000}{24 + x}(1.60)$

c. $1000 - \frac{15,000}{24 + x}(1.60)$

37. **Open-Ended** Write a rational equation that has the same solution as the question in the cartoon.

37. Check students' work.

The Family Circus *by Bil K*

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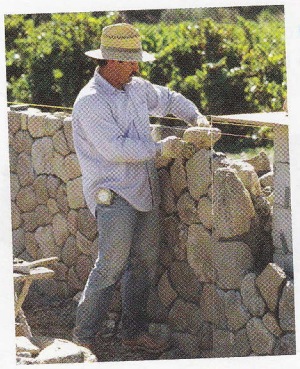
"What's 129 divided b

38. **Industry** The average hourly wage $H(x)$ of workers in an industry is modeled by the function $H(x) = \frac{16.24x}{0.062x + 39.42}$, where x represents the number of years since 1970. about 2037
   a. In what year does the model predict that wages will be $25/h?
   b. **Critical Thinking** Is the prediction reasonable? Explain. Check students' work.

Solve each equation. Check each solution.

39. $\frac{15}{x} + \frac{9x - 7}{x + 2} = 9$ 3
40. $\frac{2}{x + 2} - \frac{1}{x} = \frac{-4}{x(x + 2)}$ no
41. $\frac{1}{b + 1} + \frac{1}{b - 1} = \frac{2}{b^2 - 1}$ no solution
42. $c - \frac{c}{3} + \frac{c}{5} = 26$ 30
43. $\frac{2}{x - 3} - \frac{4}{x + 3} = \frac{8}{x^2 - 9}$ 5
44. $\frac{1}{8} + \frac{5x}{x + 2} = \frac{5}{2}$ $\frac{38}{21}$
45. $\frac{1}{x - 5} = \frac{x}{x^2 - 25}$ no solution
46. $\frac{k}{k + 1} + \frac{k}{k - 2} = 2$ -4
47. $\frac{3}{x + 5} + \frac{2}{5 - x} = \frac{-4}{x^2 - 25}$ 21
48. $\frac{5}{x + 2} = \frac{-1}{x^2 + 7x + 10} +$
49. $\frac{5}{x^2 - 7x + 12} - \frac{2}{3 - x} = \frac{5}{x - 4}$
50. $\frac{10}{2y + 8} - \frac{7y + 8}{y^2 - 16} = \frac{-8}{2y -}$
51. $\frac{7x + 3}{x^2 - 8x + 15} + \frac{3x}{x - 5} = \frac{1}{3 - x}$ $1, -\frac{2}{3}$
52. $\frac{2}{x + 3} - \frac{3}{4 - x} = \frac{2x -}{x^2 - x}$

49. no solution



Real-World Connection

Careers Masons build patterns with brick or stone.

53. **Landscape Design** Suppose you want to double the area of the patio shown at the right. Find the increase x of both the length and width of the patio. $x \approx 4.5$ ft

54. **Writing** Write and solve a problem that can be modeled by a rational equation. Check students' work.
55. **Transportation** A plane flies from New York to Chicago (about 700 miles) at a speed of 360 mi/h.
   a. The speed s of the plane is given by $s = \frac{d}{t}$. $t = \frac{d}{s}$ d represents the distance and t is the time. Solve the equation for
   b. Find the time for the trip. $\frac{35}{18}$ h
   c. On the return trip from Chicago to New York, a tail wind helps th move faster. Write an expression for the speed of the plane on th trip. Let x represent the speed of the tail wind. $\frac{d}{t} + x$
   d. The total flying time for the round trip is 3.5 h. Write a rational e for the sum of the flying times. Find the speed x of the tail wind. $\frac{700}{360} + \frac{700}{360 + x} = 3.5;$

Challenge

c. Check students' work.

56. **Open-Ended** Write a rational equation that has the following.
a. one solution **b.** two solutions **c.** no real solution

57. A salesman drove from his home to a nearby city at an average speed of 40 mi/h. He returned home at an average speed of 50 mi/h. What was his average speed for the entire trip? **44.44 mi/h**

58. An automatic pitching machine can pitch all its baseballs in $1\frac{1}{4}$ hours. One attendant can retrieve all the baseballs pitched by one machine in $3\frac{1}{2}$ hours. At least how many attendants working at the same rate should be hired so that the baseballs from 10 machines are all retrieved in less than 8 hours? **5 attendants**

FCAT Practice

Multiple Choice

59. Which value of x would NOT make the equation $\frac{5}{2x-1} = \frac{7x}{x^2-25}$ undefined? **B**
A. -5 B. 0 C. $\frac{1}{2}$ D. 5

60. What is the solution of $x + \frac{1}{x} = -2$? **I**
F. 1 or -1 G. 0 only H. $-\frac{1}{2}$ only I. -1 only

61. Which equation has 2 as an extraneous solution? **D**
A. $\frac{x+1}{17} = \frac{x+3}{15}$ B. $\frac{3}{3x+6} = \frac{4}{x^2-4}$
C. $\frac{3x+1}{3x} = \frac{5x}{5x+3}$ D. $\frac{4}{2x-4} = \frac{1}{x-2}$

FCAT Online
FCAT Format quiz at www.PHSchool.com
Web Code: aga-0906

62. Solve $\frac{2}{x+7} = \frac{x}{x^2-49}$. **F**
F. 14 only G. 7 only H. 7 or -7 I. -7 only

Short Response

63. A large snowplow can clear a parking lot in 4 hours. A small snowplow needs more time to clear the lot. Working together, they can clear the lot in 3 hours. How long would it take the small plow to clear the lot by itself? Show your work. **See margin.**

Extended Response

64. Solve and check the equation $\frac{x}{3x+9} = \frac{x+2}{x+3}$. Show your work. **See margin.**

Mixed Review

Lesson 9-5 **Simplify each difference.** **65–67. See margin.**

65. $\frac{3y+1}{4y+4} - \frac{2y+7}{2y+2}$ 66. $\frac{5x}{2y+4} - \frac{6}{y^2+2y}$ 67. $\frac{x+1}{2x-2} - \frac{2x}{x^2+2x-3}$

Lesson 8-5 **Solve each equation.**

68. $\log_{10} 0.001 = x$ **$x = -3$** 69. $\log_3 27 = 3x + 6$ **$x = -1$**

70. $\log_{0.1}(x+1) = 3$ **$x = -0.999$** 71. $\log_3 \frac{1}{9} = \frac{x}{3}$ **$x = -6$**

Lesson 7-7 **Find the inverse of each function. Is the inverse a function?**

72. $y = 5 - 2x$ **$y = \frac{5-x}{2}$; yes**
73. $y = x^2 + 1$ **$y = \pm\sqrt{x-1}$; no**
74. $y = x^3 - 4$ **$y = \sqrt[3]{x+4}$; yes**

FCAT Practice

Resources
For additional practice with a variety of test item formats:
- FCAT Practice, p. 531
- FCAT Strategies, p. 526
- FCAT Daily Practice and Strategies Transparencies

pages 514–517 Exercises

63. [2] $\frac{1}{4} + \frac{1}{x} = \frac{1}{3}$
$12x(\frac{1}{4} + \frac{1}{x}) = 12x(\frac{1}{3})$
$\frac{12x}{4} + \frac{12x}{x} = \frac{12x}{3}$
$3x + 12 = 4x$
$12 = x$
It would take the small plow 12 hours to clear the lot by itself.

[1] answer only, with no work shown

64. [4] $\frac{x}{3x+9} = \frac{x+2}{x+3}$
$\frac{x}{3(x+3)} = \frac{x+2}{x+3}$
$x(x+3) = 3(x+3)(x+2)$
$x = 3(x+2)$
$x = 3x + 6$
$-2x = 6$
$x = -3$
However, $x = -3$ makes the denominators in the original equation equal 0, which is undefined. So, the equation has no solution.

[3] one computational error

[2] gets $x = -3$, but does not state that the equation has no solution

[1] answer only, with no work shown

65. $\frac{-y-13}{4(y+1)}$

66. $\frac{5xy-12}{2y(y+2)}$

67. $\frac{x^2+3}{(2x-2)(x+3)}$