Chapter Review

ocabulary

gmented matrix (p. 218)	**matrix** (p. 164)	**rotation** (p. 190)
nter of rotation (p. 190)	**matrix addition** (p. 170)	**row operations** (p. 219)
fficient matrix (p. 210)	**matrix element** (p. 165)	**scalar** (p. 178)
nstant matrix (p. 210)	**matrix equation** (p. 172)	**scalar product** (p. 178)
amer's Rule (p. 217)	**matrix multiplication** (p. 180)	**square matrix** (p. 195)
terminant (p. 196)	**multiplicative identity matrix** (p. 195)	**transformation** (p. 188)
lation (p. 188)	**multiplicative inverse matrix** (p. 195)	**translation** (p. 188)
ual matrices (p. 173)	**preimage** (p. 188)	**variable matrix** (p. 210)
age (p. 188)	**reflection** (p. 189)	**zero matrix** (p. 171)

Reading Math
Understanding Vocabulary

Choose the correct vocabulary term to complete each sentence.

1. A _?_ is a rectangular array of numbers. **matrix**
2. Translations, dilations, reflections and rotations are all _?_. **transformations**
3. Cramer's Rule uses _?_ to solve a system of equations. **determinants**
4. If corresponding elements of matrices are equal, the matrices are _?_. **equal matrices**
5. The additive identity of a matrix is the _?_. **zero matrix**
6. A _?_ consists of a coefficient matrix, a variable matrix, and a constant matrix. **matrix equation**
7. An $n \times n$ matrix is called a _?_. **square matrix**
8. The image of a figure is a transformation of the _?_. **preimage**
9. The product of a real number and a matrix is called a _?_. **scalar product**
10. A matrix is the inverse of another matrix if their product is the _?_. **multiplicative identity matrix**

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ills and Concepts

jectives
dentify and classify
rices and their
ments (p. 164)
ganize data into
rices (p. 165)

It is often useful to organize data into matrices. A **matrix** is a rectangular array of numbers classified by its dimensions. An $m \times n$ matrix has m rows and n columns. A **matrix element** a_{ij} is in the ith row and jth column of matrix A.

State the dimensions of each matrix A. Identify the indicated element.

11. $\begin{bmatrix} 5 & 8 & -7 \\ 1 & 11 & 3 \end{bmatrix}; a_{13}$ **2 × 3; −7**

12. $\begin{bmatrix} 3 & 1 \\ -5 & 0 \\ 7 & 6 \end{bmatrix}; a_{21}$ **3 × 2; −5**

13. $\begin{bmatrix} 5 & 1 & -2 \\ 4 & -7 & 12 \\ 0 & 78 & 3 \end{bmatrix}; a_{32}$ **3 × 3; 78**

Use the matrix at the right for Exercises 14–16.

$$\begin{array}{r c} & \begin{array}{ccc} \text{1-pt Shots} & \text{2-pt Shots} & \text{3-pt Shots} \end{array} \\ \begin{array}{r} \text{Tamika} \\ \text{Johanna} \\ \text{Tran} \end{array} & \begin{bmatrix} 22 & 30 & 48 \\ 21 & 31 & 48 \\ 21 & 29 & 50 \end{bmatrix} \end{array}$$

14. How many points has Tamika scored? **226**
15. How many three-point shots has Tran made? **50**
16. What percent of Johanna's points were from one-point shots? **about 9%**

4 Chapter Review

Resources

Student Edition
Extra Practice, Ch. 4, p. 825
English/Spanish Glossary, p. 871
Properties and Formulas, p. 865
Table of Symbols, p. 861

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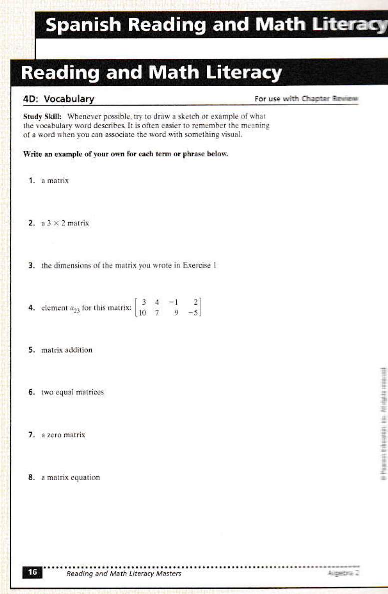
Spanish Reading and Math Literacy

Reading and Math Literacy

4D: Vocabulary — For use with Chapter Review

Study Skill: Whenever possible, try to draw a sketch or example of what the vocabulary word describes. It is often easier to remember the meaning of a word when you can associate the word with something visual.

Write an example of your own for each term or phrase below.

1. a matrix
2. a 3 × 2 matrix
3. the dimensions of the matrix you wrote in Exercise 1
4. element a_{23} for this matrix: $\begin{bmatrix} 3 & 4 & -1 & 2 \\ 10 & 7 & 9 & -5 \end{bmatrix}$
5. matrix addition
6. two equal matrices
7. a zero matrix
8. a matrix equation

16 Reading and Math Literacy Masters — Algebra 2

pages 225–227 Chapter Review

23. $\begin{bmatrix} 18 & 3 & 0 & 24 \\ -12 & 9 & 21 & 33 \end{bmatrix}$

24. $\begin{bmatrix} -9 & 7 \\ -8 & -8 \end{bmatrix}$

25. does not exist

26. $\begin{bmatrix} -6 & 10 & 21 & 41 \\ -28 & 10 & 28 & 28 \end{bmatrix}$

27. $\begin{bmatrix} -14 & -2 \\ 43 & -7 \end{bmatrix}$

28. $\begin{bmatrix} 4 & -1 & 2 \\ -1 & -2 & 3 \end{bmatrix}$

29. $\begin{bmatrix} 0 & -5 & -2 \\ 5 & 4 & 9 \end{bmatrix}$

30. $\begin{bmatrix} -3 & 2 & -1 \\ 1 & 0 & 5 \end{bmatrix}$

31. $\begin{bmatrix} 1 & 0 & 5 \\ 3 & -2 & 1 \end{bmatrix}$

32. $\begin{bmatrix} 1 & 0 & 5 \\ -3 & 2 & -1 \end{bmatrix}$

33. $\begin{bmatrix} 1.5 & -1 & 0.5 \\ 0.5 & 0 & 2.5 \end{bmatrix}$

34. $\begin{bmatrix} 6 & -4 & 2 \\ 2 & 0 & 10 \end{bmatrix}$

35. $\begin{bmatrix} -1 & 0 & -5 \\ 3 & -2 & 1 \end{bmatrix}$

4-2 and 4-3 Objectives

- To add and subtract matrices (p. 170)
- To solve some matrix equations (p. 172)
- To multiply a matrix by a scalar (p. 178)
- To multiply two matrices (p. 180)

To perform **matrix addition** or subtraction, add or subtract the corresponding elements in the matrices. To obtain the **scalar product** of a matrix and a **scalar,** multiply each matrix element by the scalar. **Matrix multiplication** uses both multiplication and addition. The element in the ith row and jth column of the product of two matrices is the sum of the products of each element of the ith ro of the first matrix and each element of the jth column of the second matrix. The first matrix must have the same number of columns as the second has rows.

Two matrices are **equal matrices** when corresponding elements are equal and t have the same dimensions. This principle is used to solve a **matrix equation.**

Solve each matrix equation for matrix X.

17. $[2 \quad -6 \quad 8] + [-1 \quad -2 \quad 4] = X$ $[1 \quad -8 \quad 12]$

18. $\begin{bmatrix} t \\ 6 \end{bmatrix} - \begin{bmatrix} 1 \\ 3 \end{bmatrix} = X$ $\begin{bmatrix} t-1 \\ 3 \end{bmatrix}$

19. $\begin{bmatrix} 7 & -1 \\ 0 & 8 \end{bmatrix} + X = \begin{bmatrix} 4 & 9 \\ -3 & 11 \end{bmatrix}$ $\begin{bmatrix} -3 & 10 \\ -3 & 3 \end{bmatrix}$

20. $X - \begin{bmatrix} -7 & 13 & 5 \\ 31 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 9 & -5 \\ 2 & 0 \end{bmatrix}$ $\begin{bmatrix} 2 & 8 & 13 \\ 33 & 0 & -7 \end{bmatrix}$

Solve for each variable.

21. $\begin{bmatrix} x-5 & 9 \\ 4 & t+2 \end{bmatrix} = \begin{bmatrix} -7 & w+1 \\ 8-r & 1 \end{bmatrix}$ $x = -2, w = 8, r = 4, t = -1$

22. $\begin{bmatrix} -4+t & 2y \\ r & w+4 \end{bmatrix} = \begin{bmatrix} 2t \\ -2r+12 \end{bmatrix}$ $t = -4, y = \frac{11}{2}, r = 4, w = 5$

Use matrices A, B, C, and D. Find each scalar product, sum, or difference, if possible. If an operation is not defined, label it *undefined.* 23–27. See margin.

$A = \begin{bmatrix} 6 & 1 & 0 & 8 \\ -4 & 3 & 7 & 11 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$ $C = \begin{bmatrix} -2 & 1 \\ 4 & 0 \\ 2 & 2 \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 5 & -2 \\ 3 & 6 \end{bmatrix}$

23. $3A$ **24.** $B - 2D$ **25.** AB **26.** BA **27.** $AC -$

4-4 Objectives

- To represent translations and dilations with matrices (p. 187)
- To represent reflections and rotations with matrices (p. 189)

A **transformation** is a change made to a figure. The original figure is the **preima** and the transformed figure is the **image.** A **translation** slides a figure without changing its size or shape. A **dilation** changes the size of a figure. You can use matrix addition to translate a figure and scalar multiplication to dilate a figure.

You can use multiplication by the appropriate matrix to perform transformatio that are specific **reflections** or **rotations.** For example, to reflect a figure in the y-axis, multiply by $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. To rotate a figure 180°, multiply by $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

For Exercises 28–35, use $\triangle ABC$ with vertices $A(3, 1)$, $B(-2, 0)$, and $C(1, 5)$. Wr the coordinates of each image in matrix form. 28–35. See margin.

28. a translation 1 unit right and 2 units down

29. a translation 3 units left and 4 units up

30. a reflection in the y-axis

31. a reflection in the line $y = x$

32. a rotation of 270°

33. a dilation half the original size

34. a dilation twice the original size

35. a rotation of 90°

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3 matrices (p. 202)

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es (p. 210)

A **square matrix** with 1's along its main diagonal and 0's elsewhere is the **multiplicative identity matrix,** I. If A and X are square matrices such that $AX = I$, then X is the **multiplicative inverse matrix** of A, A^{-1}.

You can use formulas to evaluate the determinants of 2×2 and 3×3 matrices.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \qquad \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_1b_3c_2 - a_2b_1c_3 - a_3b_2c_1$$

You can use a calculator to find the inverse of a matrix. The inverse of a 2×2 matrix can be found by using its determinant. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

You can use inverse matrices to solve some matrix equations.

You can also use inverse matrices to solve some systems of equations. When equations in a system are in standard form, the product of the **coefficient matrix** and the **variable matrix** equals the **constant matrix.** You solve the equation by multiplying both sides of the equation by the inverse of the coefficient matrix. If that inverse does not exist, the system does not have a unique solution.

Evaluate the determinant of each matrix, and find the inverse, if possible.
36–39. See margin.

36. $\begin{bmatrix} 6 & 1 \\ 0 & 4 \end{bmatrix}$ **37.** $\begin{bmatrix} 5 & -2 \\ 10 & -4 \end{bmatrix}$ **38.** $\begin{bmatrix} 10 & 1 \\ 8 & 5 \end{bmatrix}$ **39.** $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 0 & 1 \\ -1 & -2 & 0 \end{bmatrix}$

Use an inverse matrix to solve each equation or system. 40–43. See margin.

40. $\begin{bmatrix} 3 & 5 \\ 6 & 2 \end{bmatrix} X = \begin{bmatrix} -2 & 6 \\ 4 & 12 \end{bmatrix}$ **41.** $\begin{cases} x - y = 3 \\ 2x - y = -1 \end{cases}$ **42.** $\begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$

43. $\begin{bmatrix} -6 & 0 \\ 7 & 1 \end{bmatrix} X = \begin{bmatrix} -12 & -6 \\ 17 & 9 \end{bmatrix}$ **44.** $\begin{cases} x + 2y = 15 \\ 2x + 4y = 30 \end{cases}$ **45.** $\begin{cases} a + 2b + c = 14 \\ b = c + 1 \\ a = -3c + 6 \end{cases}$

no unique solution no unique solution

46. Physical Fitness A club of 17 students is going on a canoe trip. The group of people on the trip includes 5 chaperones, one for each canoe. Some canoes hold 5 people, while some hold 4 people. How many of each kind of canoe should the group rent? 3 small canoes, 2 large canoes

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Cramer's Rule for solving systems of equations uses determinants to solve for each variable. D is the determinant of the coefficient matrix. D_y is the determinant formed by replacing the coefficients of y in D with the constant terms.

You can also use **row operations** on an augmented matrix to solve a system.

Solve each system using Cramer's Rule. Check your answers by solving each system using an augmented matrix.

47. $\begin{cases} 2x - y = 15 \\ x + 3y = -17 \end{cases}$ (4, −7)

48. $\begin{cases} 3r + s - 2t = 22 \\ r + 5s + t = 4 \\ r = -3t \end{cases}$ (6, 0, −2)

Alternative Assessment, Form C

Alternative Assessment Form C

Chapter 4

Give complete answers.

TASK 1

a. Write a 3×3 matrix A. Discuss the properties that need to exist so that matrices can be added, subtracted, and multiplied.

b. Add your matrix to $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

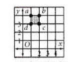
c. Subtract $\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$ from your original matrix.

d. Multiply your matrix by $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. What are the dimensions of the product matrix?

e. Find C such that $C - \begin{bmatrix} 3 & 1 & 2 \\ 6 & 4 & 5 \\ 4 & 2 & 1 \end{bmatrix} = A$, your original matrix.

f. Write a 2×2 matrix. Find the inverse of this matrix. If no inverse exists, explain why.

TASK 2



a. Organize the x- and y-coordinates of the vertices of the shaded square into a 2×4 matrix.

b. What matrix will translate the square 2 units left and 1 unit down?

c. What matrix operation is necessary to perform this translation?

d. What are the vertices of the new square?

e. Find the coordinates of the image of the original square after a dilation of 2.

Algebra 2 Chapter 4 Form C Test 35

36. 24; $\begin{bmatrix} \frac{1}{6} & -\frac{1}{24} \\ 0 & \frac{1}{4} \end{bmatrix}$

37. 0; does not exist

38. 42; $\begin{bmatrix} \frac{5}{42} & -\frac{1}{42} \\ -\frac{4}{21} & \frac{5}{21} \end{bmatrix}$

39. 6; $\begin{bmatrix} \frac{1}{3} & -\frac{2}{3} & 0 \\ -\frac{1}{6} & \frac{1}{3} & -\frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & 0 \end{bmatrix}$

40. $\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$

41. (−4, −7)

42. $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

43. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

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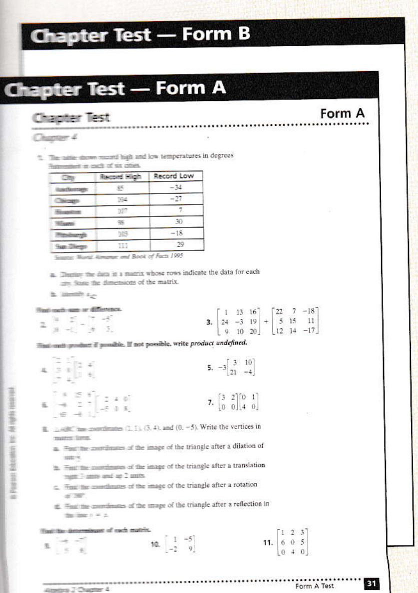

Chapter 4 Chapter Test

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History Use the table below for Exercises 1–3.

President	Years	Vetoes	Overrides
Kennedy	3	21	9
Johnson	5	30	0
Nixon	5.5	43	7
Ford	2.5	66	12
Carter	4	31	2
Reagan	8	78	9
G.H.W. Bush	4	46	1
Clinton	8	38	2

SOURCE: Congressional Research Service.
Go to **www.PHSchool.com** for a data update.
Web Code: agg-2041

1. Display the data in a matrix in which each row represents a president. **See back of book.**
2. State the dimensions of the matrix. **8 × 3**
3. Find and identify a_{32}. **43, the number of Nixon's vetoes**

Find each sum or difference.

4. $\begin{bmatrix} 4 & 7 \\ -2 & 1 \end{bmatrix} - \begin{bmatrix} -9 & 3 \\ 6 & 0 \end{bmatrix}$ $\begin{bmatrix} 13 & 4 \\ -8 & 1 \end{bmatrix}$
5. $\begin{bmatrix} 4 & -5 & 1 \\ 10 & 7 & 4 \\ 21 & -9 & -6 \end{bmatrix} + \begin{bmatrix} -7 & -10 & 4 \\ 17 & 0 & 3 \\ -2 & -6 & 1 \end{bmatrix}$ **See margin.**

Find each product. **6–8. See margin.**

6. $\begin{bmatrix} 2 & 6 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 \\ 3 & 1 \end{bmatrix}$
7. $2\begin{bmatrix} -8 & 5 & -1 \\ 0 & 9 & 7 \end{bmatrix}$
8. $\begin{bmatrix} 0 & 3 \\ -4 & 9 \end{bmatrix}\begin{bmatrix} -4 & 6 & 1 & 3 \\ 9 & -8 & 10 & 7 \end{bmatrix}$

Parallelogram $ABCD$ has coordinates $A(2, -1)$, $B(4, 3)$, $C(1, 5)$, and $D(-1, 1)$. Write a matrix for the vertices of its image after each transformation.

9. a dilation of size $\frac{2}{3}$ **See margin.**
10. a translation right 2 units and down 4 units
11. a reflection in $y = x$
12. a rotation of 270°
13. Graph parallelogram $ABCD$ and its image from Question 11 on the same coordinate plane.

10–14. See back of book.

14. **Open-Ended** Write a matrix that has no inverse.
15. **Writing** Explain how to determine whether two matrices can be multiplied and what the dimensions of the product matrix will be. **See back of book.**
16. Find the value of each variable.
$\begin{bmatrix} x & 1 & y \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-x & z & 2 \\ 2+w & 4-t & 1 \end{bmatrix}$
$x = \frac{1}{2}, z = 1, y = 2, w = 0, t = 4$

Find the determinant of each matrix.

17. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **1**
18. $\begin{bmatrix} 2 & 3 & 0 \\ -1 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ **5**
19. $\begin{bmatrix} 8 & -3 \\ 2 & 9 \end{bmatrix}$ **7**
20. $\begin{bmatrix} 0 & 3 \\ x & t \end{bmatrix}$ **$-3x$**
21. $\begin{bmatrix} \frac{1}{2} & -1 \\ 3 & 0 \end{bmatrix}$ **3**

22–26. See back of book.

Find the inverse of each matrix, if it exists.

22. $\begin{bmatrix} 3 & 8 \\ -7 & 10 \end{bmatrix}$
23. $\begin{bmatrix} 0 & -5 \\ 9 & 6 \end{bmatrix}$
24. $\begin{bmatrix} \frac{1}{2} & -1 \\ 0 & 4 \end{bmatrix}$
25. $\begin{bmatrix} -8 & 4 & -11 \\ 5 & 2 & 9 \\ -5 & 6 & 2 \end{bmatrix}$
26. $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 2 & 1 & 1 \end{bmatrix}$

Solve each matrix equation.

27. $\begin{bmatrix} 3 & -8 \\ 10 & 5 \end{bmatrix} - X = \begin{bmatrix} 2 & 8 \\ -1 & 12 \end{bmatrix}$ $\begin{bmatrix} 1 & -16 \\ 11 & -7 \end{bmatrix}$
28. $\begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}X = \begin{bmatrix} -10 & -11 \\ 26 & -36 \end{bmatrix}$ $\begin{bmatrix} -6 & 1 \\ 4 & -7 \end{bmatrix}$
29. $2X - \begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ -15 & 9 \end{bmatrix}$ $\begin{bmatrix} \frac{3}{2} & 5 \\ -7 & \frac{13}{2} \end{bmatrix}$

Solve each system using inverse matrices.

30. $\begin{cases} 2x - y = 5 \\ x + 4y = 7 \end{cases}$ **(3, 1)**
31. $\begin{cases} x + 2y + z = \ldots \\ 4x - y - z = \ldots \\ 2z = \ldots \end{cases}$ **(0, −2**
32. Solve $\begin{cases} -2x + 7y = 19 \\ x + 3y = 10 \end{cases}$ using Cramer's Rule. **(1, 3)**
33. Solve the system using an augmented matrix. **(7, −8, 10)**
$\begin{cases} x + y + z = 9 \\ 4x + 3y - z = -6 \\ -x - y + 2z = 21 \end{cases}$

page 228 Chapter Test

5. $\begin{bmatrix} -3 & -15 & 5 \\ 27 & 7 & 7 \\ 19 & -15 & -5 \end{bmatrix}$
6. $\begin{bmatrix} 16 & 16 \\ -1 & 5 \end{bmatrix}$
7. $\begin{bmatrix} -16 & 10 & -2 \\ 0 & 18 & 14 \end{bmatrix}$
8. $\begin{bmatrix} 27 & -24 & 30 & 21 \\ 97 & -96 & 86 & 51 \end{bmatrix}$
9. $\begin{bmatrix} \frac{4}{3} & \frac{8}{3} & \frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & 2 & \frac{10}{3} & \frac{2}{3} \end{bmatrix}$