Annuities

You normally don't save by depositing 1 amount and leaving it.

Annuity a series of equal payments made at equal time periods.

Example John had a 401 K plan at work. He deposits $100 from his paycheck every month. This has been happening for 7 years.

Future value of an annuity

$$A = \frac{P\left[\left(1 + \frac{r}{n}\right)^{(nt)} - 1\right]}{\left(\frac{r}{n}\right)}$$

Example How much is John's 401k worth after 7 years if he earned 5% compounded interest on his $100 per month deposits compounded monthly.

A = Future value

P = the amount you deposit EACH period

r = rate in decimal form

t = years or fraction of a year

n = # times per year rate is compounded

$$A = P\frac{\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\left(\frac{r}{n}\right)} \qquad \frac{r}{n} = .004166$$

$$A = 100\frac{\left[\left(1+\frac{0.05}{12}\right)^{12\cdot 7}-1\right]}{\left(\frac{0.05}{12}\right)} \quad \text{(A)}$$

$$\overset{\text{(B)}}{=} \frac{100(1.00417)^{84}-1}{0.00417} \overset{\text{(C)}}{=} \frac{41.80}{0.00417} \approx 10,024.85$$

Present value of an annuity

$$P = \frac{A\left(\frac{r}{n}\right)}{\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}$$

$$\frac{2{,}000{,}000\left(\frac{.08}{12}\right)}{\left[\left(1+\frac{.08}{12}\right)^{(12\cdot 50)} - 1\right]}$$

Example I want $2 million when I retire in 50 years. What amount must I deposit into my IRA each month? I hope to earn a return of 8% compounded monthly

$$2{,}000{,}000 = \frac{P\left[\left(1+\frac{.08}{12}\right)^{12\cdot 50}-1\right]}{\left(\frac{.08}{12}\right)}$$

$$2{,}000{,}000 = \frac{P(53.878-1)}{0.00667}$$

$$\frac{2{,}000{,}000(0.00667)}{52.878} = P$$

Deposit $252.15 per month

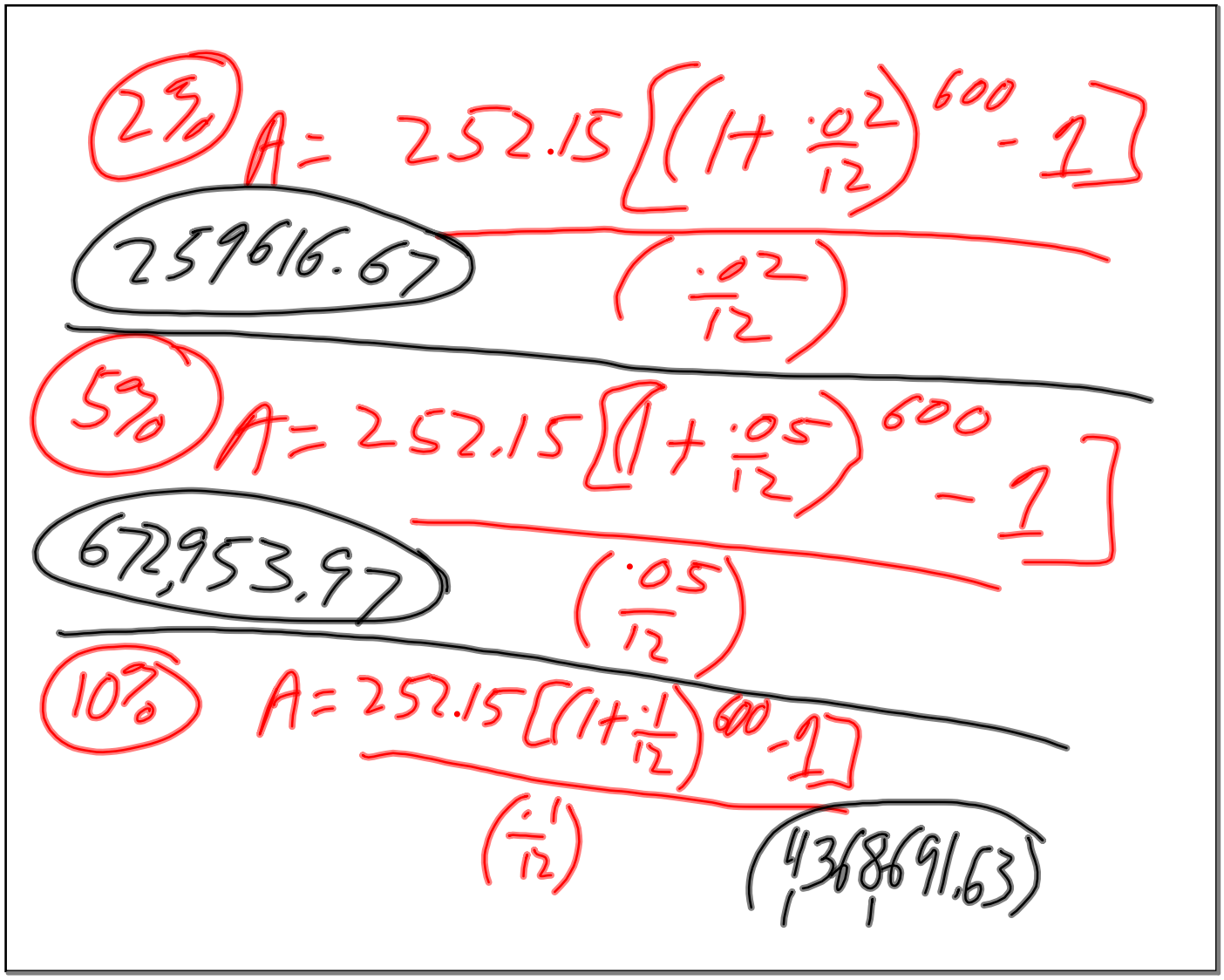
Explain what your answer means in words.

I must deposit $252.15 per month for 50 years in order to get $2 million. AS LONG AS I earn 8%.

What is the future value of that $252.15 per month if I earn 2%?

If I earn 5%?

If I earn 10%?


2%
A = 252.15 [(1 + .02/12)^600 - 1]
(.02/12)
259616.67
5%
A = 252.15 [(1 + .05/12)^600 - 1]
(.05/12)
672,953.97
10%
A = 252.15 [(1 + .1/12)^600 - 1]
(.1/12)
1,368,691.63

What is the amount of money I have to deposit each month if I want $2 million in 50 years, compounded monthly.

Use the following rates:

@ 2%

ⓑ 5%

© 10%

2%

$$2{,}000{,}000 = P\frac{\left[\left(1+\frac{.02}{12}\right)^{(600)} - 1\right]}{\left(\frac{.02}{12}\right)}$$

$1942.48

5%

$$2{,}000{,}000 = P\frac{\left[\left(1+\frac{.05}{12}\right)^{600} - 1\right]}{\left(\frac{.05}{12}\right)}$$

$749.44

10%

$$2{,}000{,}000 = P\frac{\left[\left(1+\frac{.1}{12}\right)^{600} - 1\right]}{\left(\frac{.1}{12}\right)}$$

$1115.44

Calculate the future value of the $252.17 monthly deposit if you did it for 3, 5, 10, 15, 20 and 30 years. Rate = 8%

3, 5, 10, 15, 20, 30 yrs

(3) $A = 252.17\left[\left(1+\frac{.08}{12}\right)^{36}-1\right] \div \left(\frac{.08}{12}\right)$

$10221.85

(5) $A = 252.17\left[\left(1+\frac{.08}{12}\right)^{60}-1\right] \div \left(\frac{.08}{12}\right)$

$18,528.66

(10) $A = \dfrac{252.17\left[\left(1+\frac{.08}{12}\right)^{120}-1\right]}{\left(\frac{.08}{12}\right)}$

$46,133.50

(15) $A = \dfrac{252.17\left[\left(1+\frac{.08}{12}\right)^{180}-1\right]}{\left(\frac{.08}{12}\right)}$

$87,260.46

(20) $A = \dfrac{252.17\left[\left(1+\frac{.08}{12}\right)^{240} - 1\right]}{\left(\frac{.08}{12}\right)}$

$148,533.28

(30) $A = \dfrac{252.17\left[\left(1+\frac{.08}{12}\right)^{360} - 1\right]}{\left(\frac{.08}{12}\right)}$

$375,823.94

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