

C LENGTH

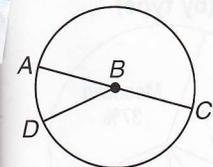
Teaching Tip Tell students that they set up a proportion to find an arc length because they are finding a portion of the circumference. Explain that this process is very similar to finding a percent of a whole.

Class Example



Teaching Tip If students want to see this problem another way, explain that 120 is $\frac{1}{3}$ of 360 as each arc length would be equal to $\frac{1}{3}$ of the total circumference. So, students can divide 30π by 3 and get the same answer.

In $\odot B$, $AC = 9$ and $m\angle ABD = 40$. Find the length of \widehat{AD} .



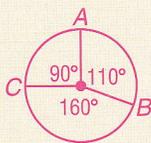
10π units or about 31.42 units

Teaching Tip Have students construct a circle like the one in Example 4 and measure its radius. Have them use string to trace the circumference of the circle. Mark on the string the points that are the endpoints of the arc. After calculating the circumference, have them use a ruler to verify the arc length.

Answers

Sample answer:

\widehat{AB} , \widehat{BC} , \widehat{AC} , \widehat{ABC} , \widehat{BCA} , \widehat{CAB} ; $m\widehat{AB} = 110$, $m\widehat{BC} = 160$, $m\widehat{AC} = 90$, $m\widehat{ABC} = 270$, $m\widehat{BCA} = 250$, $m\widehat{CAB} = 200$



A diameter divides the circle into two congruent arcs. Without the third letter, it is impossible to know which semicircle is being referenced.

Sample answer: Concentric circles have the same center, but different radius measures; congruent circles usually have different centers but the same radius measure.

ARC LENGTH Another way to measure an arc is by its length. An arc is part of the circle, so the length of an arc is a part of the circumference.

Study Tip

Look Back

To review proportions, see Lesson 6-1.

Example 4 Arc Length

In $\odot P$, $PR = 15$ and $m\angle QPR = 120$. Find the length of \widehat{QR} .

In $\odot P$, $r = 15$, so $C = 2\pi(15)$ or 30π and $m\widehat{QR} = m\angle QPR$ or 120. Write a proportion to compare each part to its whole.

$$\frac{\text{degree measure of arc}}{\text{degree measure of whole circle}} \rightarrow \frac{120}{360} = \frac{\ell}{30\pi} \leftarrow \begin{array}{l} \text{arc length} \\ \text{circumference} \end{array}$$

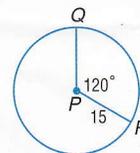
Now solve the proportion for ℓ .

$$\frac{120}{360} = \frac{\ell}{30\pi}$$

$$\frac{120}{360}(30\pi) = \ell \quad \text{Multiply each side by } 30\pi.$$

$$10\pi = \ell \quad \text{Simplify.}$$

The length of \widehat{QR} is 10π units or about 31.42 units.



The proportion used to find the arc length in Example 4 can be adapted to find the arc length in any circle.

Key Concept

Arc Length

$$\frac{\text{degree measure of arc}}{\text{degree measure of whole circle}} \rightarrow \frac{A}{360} = \frac{\ell}{2\pi r} \leftarrow \begin{array}{l} \text{arc length} \\ \text{circumference} \end{array}$$

$$\text{This can also be expressed as } \frac{A}{360} \cdot C = \ell.$$

Check for Understanding

Concept Check

- OPEN-ENDED** Draw a circle and locate three points on the circle. Name all the arcs determined by the three points and use a protractor to find the measure of each arc. **1-3. See margin.**
- Explain why it is necessary to use three letters to name a semicircle.
- Describe the difference between *concentric* circles and *congruent* circles.

Guided Practice

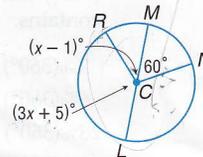
ALGEBRA Find each measure.

4. $m\angle NCL$ **120**

6. $m\angle RCM$ **43**

5. $m\angle RCL$ **137**

7. $m\angle RCN$ **103**



GUIDED PRACTICE KEY	
Exercises	Examples
4-7	1
8-11	2
12	3
13	4

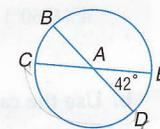
In $\odot A$, $m\angle EAD = 42$. Find each measure.

8. $m\widehat{BC}$ **42**

10. $m\widehat{EDB}$ **222**

9. $m\widehat{CBE}$ **180**

11. $m\widehat{CD}$ **138**



12. Points T and R lie on $\odot W$ so that $WR = 12$ and $m\angle TWR = 60$. Find the length of \widehat{TR} . **$4\pi \approx 12.57$ units**

DAILY

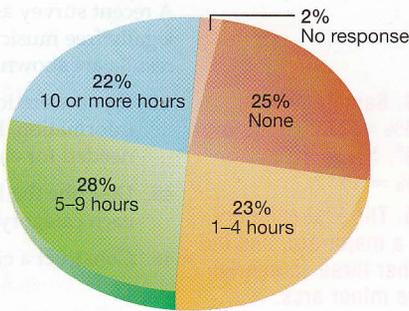
INTERVENTION

Unlocking Misconceptions

Arcs Students may sometimes confuse the terms *arc measure* and *arc length*. Explain that they can remember that angles have degree measure, denoted $m\angle ABC$; similarly, arcs have degree measure, denoted $m\widehat{AC}$. Just as segment length is a distance along a line, arc length is a distance along a curve that you can actually follow or draw with a pencil. Point out that students should be careful to determine whether they need to find the *measure* or *length* of an arc.

Application 13. SURVEYS The graph shows the results of a survey of 1400 chief financial officers who were asked how many hours they spend working on the weekend. Determine the measurement of each angle of the graph. Round to the nearest degree. **Sample answer: 25% = 90°, 23% = 83°, 28% = 101°, 22% = 79°, 2% = 7°**

Executives Working on the Weekend



Source: Accountemps

Study Guide and Intervent p. 547 (shown) and p. 548

Angles and Arcs A central angle is an angle whose vertex is at the center of a circle and whose sides are radii. A central angle separates a circle into two arcs, a major arc and a minor arc.

Here are some properties of central angles and arcs.

- The sum of the measures of the central angles of a circle with no interior points in common is 360.
- The measure of a minor arc equals the measure of its central angle.
- The measure of a major arc is 360 minus the measure of the minor arc.
- Two arcs are congruent if and only if their corresponding central angles are congruent.
- The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. (Arc Addition Postulate)

Example In $\odot R$, $m\angle ARB = 42^\circ$ and \overline{AC} is a diameter. Find $m\widehat{AB}$ and $m\widehat{ACB}$.

$\angle ARB$ is a central angle and $m\angle ARB = 42^\circ$, so $m\widehat{AB} = 42$. Thus $m\widehat{ACB} = 360 - 42$ or 318.

EXERCISES

Find each measure.

- $m\angle SCT$ 75
- $m\angle SCU$ 135
- $m\angle SCQ$ 90
- $m\angle QCT$ 165

If $m\angle BOA = 44^\circ$, find each measure.

- $m\widehat{BA}$ 44
- $m\widehat{BC}$ 136
- $m\widehat{CB}$ 44
- $m\widehat{ACB}$ 316
- $m\widehat{BCD}$ 180
- $m\widehat{AD}$ 136

Skills Practice, p. 549 and Practice, p. 550 (shown)

ALGEBRA In $\odot Q$, \overline{AC} and \overline{BD} are diameters. Find each measure.

- $m\angle AQE$ 59
- $m\angle DQE$ 48
- $m\angle CQD$ 73
- $m\angle BQC$ 107
- $m\angle CQE$ 121
- $m\angle AQD$ 107

In $\odot P$, $m\angle GPH = 38^\circ$. Find each measure.

- $m\widehat{EP}$ 38
- $m\widehat{DE}$ 52
- $m\widehat{FG}$ 142
- $m\widehat{DHG}$ 128
- $m\widehat{DFG}$ 232
- $m\widehat{DGE}$ 308

The radius of $\odot Z$ is 13.5 units long. Find the length of each arc for the given angle measure.

- \widehat{QP} if $m\angle QZT = 120^\circ$
 $9\pi \approx 28.27$ units
- \widehat{QR} if $m\angle QZR = 60^\circ$
 $4.5\pi \approx 14.14$ units
- \widehat{PR} if $m\angle PZR = 150^\circ$
 $11.25\pi \approx 35.34$ units
- \widehat{PQ} if $m\angle PQS = 160^\circ$
 $12\pi \approx 37.70$ units

HOMEWORK For Exercises 17 and 18, refer to the table, which shows the number of hours students at Leland High School say they spend on homework each night.

17. If you were to construct a circle graph of the data, how many degrees would be allotted to each category?
28.8°, 104.4°, 208.8°, 10.8°, 7.2°

18. Describe the arcs associated with each category.
The arc associated with 2–3 hours is a major arc; minor arcs are associated with the remaining categories.

Reading to Learn Mathematics, p. 551

Pre-Activity What kinds of angles do the hands on a clock form?

- Read the introduction to Lesson 10-2 at the top of page 533.
- What is the measure of the angle formed by the hour and minute hands of the clock at 5:00? 150°
 - What is the measure of the angle formed by the hour hand at 10:30? (Hint: How has each hand moved since 10:00?)

Reading the Lesson

- Refer to $\odot P$. Indicate whether each statement is true or false.
 - \widehat{DAB} is a major arc. **false**
 - \widehat{ADC} is a semicircle. **true**
 - $\widehat{AD} = \widehat{CD}$. **true**
 - \widehat{DA} and \widehat{AB} are adjacent arcs. **true**
 - $\angle BPC$ is an acute central angle. **false**
 - $\angle DPA$ and $\angle BPA$ are supplementary central angles. **false**
- Refer to the figure in Exercise 1. Give each of the following arc measures.
 - $m\widehat{AB}$ 52
 - $m\widehat{CB}$ 90
 - $m\widehat{BC}$ 128
 - $m\widehat{ADC}$ 180
 - $m\widehat{DAB}$ 142
 - $m\widehat{DCB}$ 218
 - $m\widehat{DAC}$ 270
 - $m\widehat{BDA}$ 308
- Underline the correct word or number to form a true statement.
 - The arc measure of a semicircle is (90/180/360).
 - Arcs of a circle that have exactly one point in common are (congruent/opposite/adjacent) arcs.
 - The measure of a major arc is greater than (0/90/180) and less than 360.
 - Suppose a set of central angles of a circle have interiors that do not overlap. The sum of the measures of the central angles is (90/270/360).
 - The measure of an arc formed by two adjacent arcs is the (sum/difference) of the measures of the two arcs.
 - The measure of a minor arc is greater than (0/90/180) and less than 360.

Helping You Remember

4. A good way to remember something is to explain it to someone else. Classmate Luis does not like to work with proportions. What is a way to find the length of a minor arc of a circle without solving a proportion? **5. Divide the measure of the central angle of the arc by 360. Multiply this fraction by the circumference of the circle to find the length of the arc.**

★ indicates increased difficulty
Practice and Apply

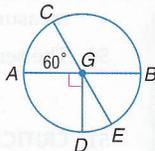
Homework Help

For Exercises	See Examples
23	1
31	2
43	3
45	4

Extra Practice See page 774.

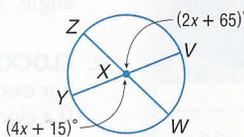
Find each measure.

- $m\angle CGB$ 120
- $m\angle AGD$ 90
- $m\angle CGD$ 150
- $m\angle BGE$ 60
- $m\angle DGE$ 30
- $m\angle AGE$ 120



ALGEBRA Find each measure.

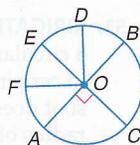
- $m\angle ZXV$ 115
- $m\angle ZXY$ 65
- $m\angle YXW$ 115
- $m\angle VXW$ 65



In $\odot O$, \overline{EC} and \overline{AB} are diameters, and $\angle BOD \cong \angle DOE \cong \angle EOF \cong \angle FOA$.

Find each measure.

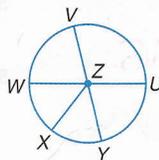
- $m\widehat{BC}$ 90
- $m\widehat{AE}$ 90
- $m\widehat{ACB}$ 180
- $m\widehat{CBF}$ 225
- $m\widehat{AC}$ 90
- $m\widehat{EB}$ 90
- $m\widehat{AD}$ 135
- $m\widehat{ADC}$ 270



ALGEBRA In $\odot Z$, $\angle WZX \cong \angle XZY$, $m\angle VZU = 4x$, $m\angle UZY = 2x + 24$, and \overline{VY} and \overline{WU} are diameters.

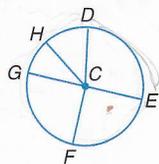
Find each measure.

- $m\widehat{UY}$ 76
- $m\widehat{WX}$ 52
- $m\widehat{WUY}$ 256
- $m\widehat{XVY}$ 308
- $m\widehat{WV}$ 76
- $m\widehat{XY}$ 52
- $m\widehat{YVW}$ 256
- $m\widehat{WUX}$ 308



The diameter of $\odot C$ is 32 units long. Find the length of each arc for the given angle measure.

- \widehat{DE} if $m\angle DCE = 100^\circ$
 $\frac{18\pi}{9} \approx 27.93$ units
- \widehat{HDF} if $m\angle HCF = 125^\circ$
 $\frac{188\pi}{9} \approx 65.62$ units
- \widehat{HE} if $m\angle DCE = 90^\circ$
 $4\pi \approx 75.40$ units
- \widehat{HD} if $m\angle DCH = 45^\circ$
 $4\pi \approx 12.57$ units



Enrichment, p. 552

Curves of Constant Width

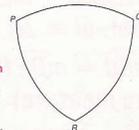
A circle is called a curve of constant width because no matter how you turn it, the greatest distance across it is always the same. However, the circle is not the only figure with this property.

The figure at the right is called a Reuleaux triangle.

- Use a metric ruler to find the distance from P to any point on the opposite side. 4.8 cm
- Find the distance from Q to the opposite side. 4.8 cm
- What is the distance from R to the opposite side? 4.8 cm

The Reuleaux triangle is made of three arcs. In the example shown, \widehat{PQ} has center R, \widehat{QR} has center P, and \widehat{PR} has center Q.

- Trace the Reuleaux triangle above on a piece of paper and cut it out. Make a square with sides the length you found in Exercise 1. Show that you can turn the triangle inside the square.



Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective

- Angles and Arcs: 14–43
- Arc Length: 44–45

Odd/Even Assignments

Exercises 14–43 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 46 requires a compass.

Assignment Guide

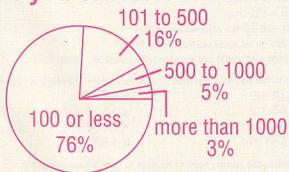
Basic: 15–37 odd, 41–55 odd, 57–76

Average: 15–55 odd, 57–76

Advanced: 14–50 even, 51, 52, 54, 55–70 (optional: 71–76)

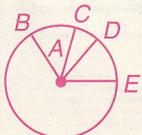
Answers

1. How many free files have you collected?



2. Given: $\angle BAC \cong \angle DAE$

Prove: $\widehat{BC} \cong \widehat{DE}$



44. **Sample answer:**
 $76\% = 273^\circ$, $16\% = 58^\circ$, $5\% = 18^\circ$,
 $3\% = 11^\circ$

45. **The first category is a major arc, and the other three categories are minor arcs.**

ONLINE MUSIC For Exercises 44–46, refer to the table and use the following information.

A recent survey asked online users how many legally free music files they have collected. The results are shown in the table.

Free Music Downloads	
How many free music files have you collected?	
100 files or less	76%
101 to 500 files	16%
501 to 1000 files	5%
More than 1000 files	3%

Source: QuickTake.com

44. If you were to construct a circle graph of this information, how many degrees would be needed for each category?

45. Describe the kind of arc associated with each category.

46. Construct a circle graph for these data. **See margin.**

Determine whether each statement is *sometimes*, *always*, or *never* true.

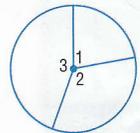
47. The measure of a major arc is greater than 180. **always**

48. The central angle of a minor arc is an acute angle. **sometimes**

49. The sum of the measures of the central angles of a circle depends on the measure of the radius. **never**

50. The semicircles of two congruent circles are congruent. **always**

51. **CRITICAL THINKING** Central angles 1, 2, and 3 have measures in the ratio 2 : 3 : 4. Find the measure of each angle. **$m\angle 1 = 80$, $m\angle 2 = 120$, $m\angle 3 = 160$**



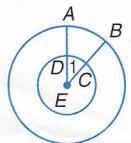
52. **CLOCKS** The hands of a clock form the same angle at various times of the day. For example, the angle formed at 2:00 is congruent to the angle formed at 10:00. If a clock has a diameter of 1 foot, what is the distance along the edge of the clock from the minute hand to the hour hand at 2:00? **2π in. \approx 6.3 in.**

53. **IRRIGATION** Some irrigation systems spray water in a circular pattern. You can adjust the nozzle to spray in certain directions. The nozzle in the diagram is set so it does not spray on the house. If the spray has a radius of 12 feet, what is the approximate length of the arc that the spray creates? **56.5 ft**



54. **PROOF** Write a proof of Theorem 10.1. **See margin.**

55. **CRITICAL THINKING** The circles at the right are concentric circles that both have point E as their center. If $m\angle 1 = 42$, determine whether $\widehat{AB} \cong \widehat{CD}$. Explain. **No; the radii are not equal, so the proportional part of the circumferences would not be the same. Thus, the arcs would not be congruent.**



56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

What kind of angles do the hands of a clock form?

Include the following in your answer:

- the kind of angle formed by the hands of a clock, and
- several times of day when these angles are congruent.

534 Chapter 10 Circles

Proof:
Statements (Reasons)

1. $\angle BAC \cong \angle DAE$ (Given)
2. $m\angle BAC = m\angle DAE$
(Def. of $\cong \angle$)
3. $m\widehat{BC} = m\widehat{DE}$ (Def. of arc measure)
4. $\widehat{BC} \cong \widehat{DE}$ (Def. of \cong arcs)

56. Sample answer: The hands of the clock form central angles. Answers should include the following.

- The hands form acute, right, and obtuse angles.
- Some times when the angles formed by the minute and hour hand are congruent are at 1:00 and 11:00, 2:00 and 10:00, 3:00 and 9:00, 4:00 and 8:00, and 5:00 and 7:00. They also form congruent angles at many other times of the day, such as 3:05 and 8:55.