You can also use other geometric figures to help you find the circumference of a circle.

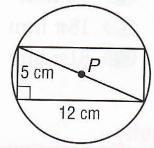
FCAT Practice
Standardized Test Practice

Example 5 Use Other Figures to Find Circumference

Multiple-Choice Test Item

Find the exact circumference of $\odot P$.

(A) 13 cm
(B) 12π cm
(C) 40.84 cm
(D) 13π cm



Read the Test Item

You are given a figure that involves a right triangle and a circle. You are asked to find the exact circumference of the circle.

Solve the Test Item

The diameter of the circle is the same as the hypotenuse of the right triangle.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$5^2 + 12^2 = c^2$ Substitution

$169 = c^2$ Simplify.

$13 = c$ Take the square root of each side.

So the diameter of the circle is 13 centimeters.

$C = \pi d$ Circumference formula

$C = \pi(13)$ or 13π Substitution

Because we want the exact circumference, the answer is D.

The Princeton Review

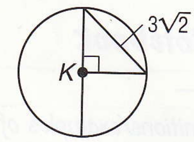
Test-Taking Tip

Notice that the problem asks for an exact answer. Since you know that an exact circumference contains π, you can eliminate choices A and C.

In-Class Example

PowerPoint

5 Find the exact circumference of $\odot K$. **B**



A $3\sqrt{2}\pi$ **B** 6π
C $6\sqrt{2}\pi$ **D** 12π

Answers

1. Sample answer: The value of π is calculated by dividing the circumference of a circle by the diameter.

3. Except for a diameter, two radii and a chord of a circle can form a triangle. The Triangle Inequality Theorem states that the sum of two sides has to be greater than the third. So, $2r$ has to be greater than the measure of any chord that is not a diameter, but $2r$ is the measure of the diameter. So the diameter has to be longer than any other chord of the circle.

Check for Understanding

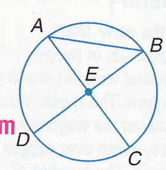
Concept Check

1. **Describe** how the value of π can be calculated. **See margin.**
2. **Write** two equations that show how the diameter of a circle is related to the radius of a circle. $d = 2r$, $r = \frac{1}{2}d$
3. **OPEN ENDED** Explain why a diameter is the longest chord of a circle. **See margin.**

Guided Practice

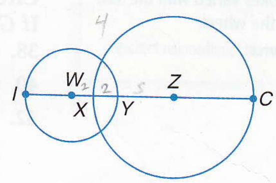
GUIDED PRACTICE KEY	
Exercises	Examples
4–7	1
8, 9	2
10–12	3
13, 14	4
15	5

For Exercises 4–9, refer to the circle at the right.



4. Name the circle. $\odot E$
5. Name a radius. $\overline{EA}$, $\overline{EB}$, $\overline{EC}$, or $\overline{ED}$
6. Name a chord. $\overline{AB}$, $\overline{AC}$, or $\overline{BD}$
7. Name a diameter. $\overline{AC}$ or $\overline{BD}$
8. Suppose $BD = 12$ millimeters. Find the radius of the circle. **6 mm**
9. Suppose $CE = 5.2$ inches. Find the diameter of the circle. **10.4 in.**

Circle W has a radius of 4 units, $\odot Z$ has a radius of 7 units, and $XY = 2$. Find each measure.



10. YZ **5**
11. IX **6**
12. IC **20**

DAILY INTERVENTION

Differentiated Instruction

ELL

Verbal/Linguistic Have students write about the parts of a circle and its circumference in their own words. They can write a paragraph that explains each vocabulary term and the relationship of the terms to each other, or they can list the terms and write a brief explanation and/or provide an example for each. Students can use these explanations for their study notebooks.

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include one circle that is labeled to demonstrate each vocabulary term in this lesson.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective

- **Parts of Circles:** 16–43
- **Circumference:** 48–52

Odd/Even Assignments

Exercises 16–55 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 17–35 odd, 39, 41, 45–49 odd, 53–63 odd, 64, 66–80 (optional: 65)

Average: 17–61 odd, 63, 64, 66–80 (optional: 65)

Advanced: 16–60 even, 61–74 (optional: 75–80)

FCAT Practice

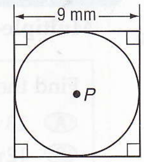
Standardized Test Practice

The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary.

13. $r = 5$ m, $d =$ _?_, $C =$ _?_ **10 m, 31.42 m**

14. $C = 2368$ ft, $d =$ _?_, $r =$ _?_ **753.76 ft, 376.88 ft**

15. Find the exact circumference of the circle. **B**



Ⓐ 4.5π mm
Ⓑ 9π mm
Ⓒ 18π mm
Ⓓ 81π mm

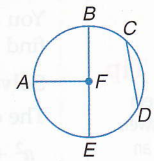
★ indicates increased difficulty

Practice and Apply

Homework Help	
For Exercises	**See Examples**
16–25	1
26–31	2
32–43	3
48–51	4
52	5

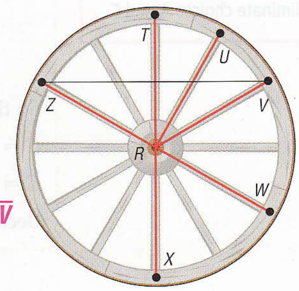
Extra Practice
See page 773.

For Exercises 16–20, refer to the circle at the right.



16. Name the circle. **⊙F**
17. Name a radius. **$\overline{FA}$, $\overline{FB}$, or $\overline{FE}$**
18. Name a chord. **$\overline{BE}$ or $\overline{CD}$**
19. Name a diameter. **$\overline{BE}$**
20. Name a radius not contained in a diameter. **$\overline{FA}$**

HISTORY For Exercises 21–31, refer to the model of the Conestoga wagon wheel.



21. Name the circle. **⊙R**
22. Name a radius of the circle. **$\overline{RT}$, $\overline{RU}$, $\overline{RV}$, $\overline{RW}$, $\overline{RX}$, or $\overline{RZ}$**
23. Name a chord of the circle. **$\overline{ZV}$, $\overline{TX}$, or $\overline{WZ}$**
24. Name a diameter of the circle. **$\overline{TX}$ or $\overline{WZ}$**
25. Name a radius not contained in a diameter. **$\overline{RU}$, $\overline{RV}$**
26. Suppose the radius of the circle is 2 feet. Find the diameter. **4 ft**
27. The larger wheel of the wagon was often 5 or more feet tall. What is the radius of a 5-foot wheel? **2.5 ft**
28. If $TX = 120$ centimeters, find TR. **60 cm**
29. If $RZ = 32$ inches, find ZW. **64 in. or 5 ft 4 in.**
30. If $UR = 18$ inches, find RV. **18 in.**
31. If $XT = 1.2$ meters, find UR. **0.6 m**

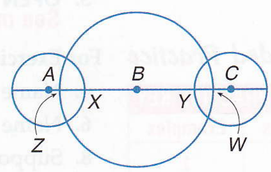
More About. . .

History

In the early 1800s, thousands of people traveled west in Conestoga wagons. The spoke wheels enabled the wagons to carry cargo over rugged terrain. The number of spokes varied with the size of the wheel.

Source: Smithsonian Institute

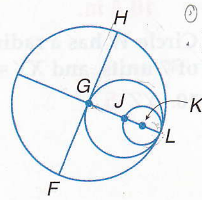
The diameters of ⊙A, ⊙B, and ⊙C are 10, 30, and 10 units, respectively. Find each measure if $\overline{AZ} \cong \overline{CW}$ and $CW = 2$.



32. AZ **2**
33. ZX **3**
34. BX **12**
35. BY **12**
36. YW **3**
★37. AC **34**

d
OJ= 10
K= 5

Circles G, J, and K all intersect at L. If $GH = 10$, find each measure.



38. FG **10**
39. FH **20**
40. GL **10**
41. GJ **5**
42. JL **5**
★43. JK **2.5**

Answer

62. Sample answer: about 251.3 feet. Answers should include the following.

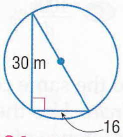
- The distance the animal travels is approximated by the circumference of the circle.
- The diameter for the circle on which the animal is located becomes $80 - 2$ or 78. The circumference of this circle is 78π. Multiply by 22 to get a total distance of $22(78\pi)$ or 5391 feet. This is a little over a mile.

44. 14 mm, 43.98 mm
45. 13.4 cm, 84.19 cm
46. 26 mi, 13 mi
47. 24.32 m, 12.16 m
48. $6\frac{1}{4}$ yd, 39.27 yd
49. $13\frac{1}{2}$ in., 42.41 in.

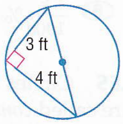
The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary.

44. $r = 7$ mm, $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$
45. $d = 26.8$ cm, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$
46. $C = 26\pi$ mi, $d = \underline{\ ?\ }$, $r = \underline{\ ?\ }$
47. $C = 76.4$ m, $d = \underline{\ ?\ }$, $r = \underline{\ ?\ }$
48. $d = 12\frac{1}{2}$yd, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$
49. $r = 6\frac{3}{4}$in., $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$
50. $d = 2a$, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$ **a, 6.28a**
★51. $r = \frac{a}{6}$, $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$ **0.33a, 1.05a**

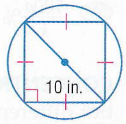
Find the exact circumference of each circle.

52. 

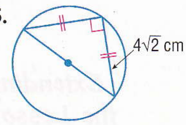
34π m

53. 

5π ft

54. 

$10\pi\sqrt{2}$ in.

55. 

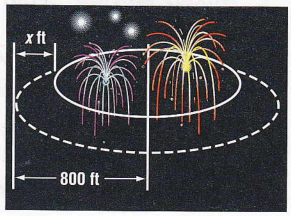
8π cm

56. 1; This description is the definition of a radius.

56. **PROBABILITY** Find the probability that a segment with endpoints that are the center of the circle and a point on the circle is a radius. Explain.

57. **PROBABILITY** Find the probability that a chord that does not contain the center of a circle is the longest chord of the circle.
0; The longest chord of a circle is the diameter, which contains the center.

FIREWORKS **For Exercises 58–60, use the following information.**
Every July 4th Boston puts on a gala with the Boston Pops Orchestra, followed by a huge fireworks display. The fireworks are shot from a barge in the river. There is an explosion circle inside which all of the fireworks will explode. Spectators sit outside a safety circle that is 800 feet from the center of the fireworks display.

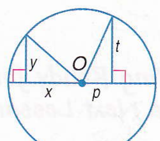


58. Find the approximate circumference of the safety circle. **5026.5 ft**
59. If the safety circle is 200 to 300 feet farther from the center than the explosion circle, find the range of values for the radius of the explosion circle. **500–600 ft**
60. Find the least and maximum circumference of the explosion circle to the nearest foot. **3142 ft; 3770 ft**

WebQuest
Drawing a radius and circle on the map is the last clue to help you find the hidden treasure. Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

Online Research **Data Update** Find the largest firework ever made. How does its dimension compare to the Boston display? Visit www.geometryonline.com/data_update to learn more.

61. **CRITICAL THINKING** In the figure, O is the center of the circle, and $x^2 + y^2 + p^2 + t^2 = 288$. What is the exact circumference of $\odot O$? **24π units**



62. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How far does a carousel animal travel in one rotation?

Include the following in your answer:
- a description of how the circumference of a circle relates to the distance traveled by the animal, and
- whether an animal located one foot from the outside edge of the carousel travels a mile when it makes 22 rotations for each ride.

www.geometryonline.com/self_check_quiz/fcat

Study Guide and Intervention, p. 541 (shown) and p. 542

Parts of Circles A **circle** consists of all points in a plane that are a given distance, called the **radius**, from a given point called the **center**.

A segment or line can intersect a circle in several ways.
- A segment with endpoints that are the center of the circle and a point of the circle is a **radius**.
- A segment with endpoints that lie on the circle is a **chord**.
- A chord that contains the circle's center is a **diameter**.

chord:
radius:
diamet

Example

a. **Name the circle.**
The name of the circle is $\odot O$.
b. **Name radii of the circle.**
$\overline{AO}$, $\overline{BO}$, $\overline{CO}$, and $\overline{DO}$ are radii.
c. **Name chords of the circle.**
$\overline{AB}$ and $\overline{CD}$ are chords.
d. **Name a diameter of the circle.**
$\overline{AB}$ is a diameter.

Exercises

1. Name the circle. **$\odot R$**
2. Name radii of the circle. **$\overline{RA}$, $\overline{RB}$, $\overline{RY}$, and $\overline{RX}$**
3. Name chords of the circle. **$\overline{BY}$, $\overline{AX}$, $\overline{AB}$, and $\overline{XY}$**
4. Name diameters of the circle. **$\overline{AB}$ and $\overline{XY}$**
5. Find AR if AB is 18 millimeters. **9 mm**
6. Find AR and AB if RY is 10 inches. **$AR = 10$ in.; $AB = 20$ in.**
7. Is $\overline{AB} \cong \overline{XY}$? Explain. **Yes; all diameters of the same circle are cong**

Skills Practice, p. 543 and Practice, p. 544 (shown)

For Exercises 1–5, refer to the circle.

1. Name the circle. **$\odot L$**
2. Name a radius. **$\overline{LR}$, $\overline{LT}$, or $\overline{LW}$**
3. Name a chord. **$\overline{RT}$, $\overline{RS}$, or $\overline{ST}$**
4. Name a diameter. **$\overline{RT}$**
5. Name a radius not drawn as part of a diameter. **$\overline{LW}$**
6. Suppose the radius of the circle is 3.5 yards. Find the diameter. **7 yd**
7. If $RT = 19$ meters, find LW. **9.5 m**

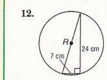
The diameters of $\odot L$ and $\odot M$ are 20 and 13 units, respectively. Find each measure if $QR = 4$.

8. LQ **6**
9. RM **2.5**

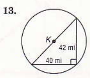
The radius, diameter, or circumference of a circle is given. Find the miss measures to the nearest hundredth.

10. $r = 7.5$ mm
$d = \underline{\ 15\text{ mm}\ }$, $C \approx \underline{\ 47.12\text{ mm}\ }$
11. $C = 227.6$ yd
$d \approx \underline{\ 72.45\text{ yd}\ }$, $r \approx \underline{\ 36.22}$

Find the exact circumference of each circle.

12. 

25π cm

13. 

58π mi

SUNDIALS **For Exercises 14 and 15, use the following information.**
Herman purchased a sundial to use as the centerpiece for a garden. The diameter sundial is 9.5 inches.

14. Find the radius of the sundial. **4.75 in.**
15. Find the circumference of the sundial to the nearest hundredth. **29.85 in.**

Reading to Learn Mathematics, p. 545

ELL

Pre-Activity **How far does a carousel animal travel in one rotation?**

Read the introduction to Lesson 10-1 at the top of page 522 in you

How could you measure the approximate distance around the circ carousel using everyday measuring devices? **Sample answer: piece of string along the rim of the carousel. Cut off a of string that covers the perimeter of the circle. Straig string and measure it with a yardstick.**

Reading the Lesson

1. Refer to the figure.
   a. Name the circle. **$\odot Q$**
   b. Name four radii of the circle. **$\overline{QP}$, $\overline{QR}$, $\overline{QS}$, and $\overline{QT}$**
   c. Name a diameter of the circle. **$\overline{PR}$**
   d. Name two chords of the circle. **$\overline{PR}$ and $\overline{ST}$**
2. Match each description from the first column with the best term from the seco column. (Some terms in the second column may be used more than once or not
   a. a segment whose endpoints are on a circle **iii**
   b. the set of all points in a plane that are the same distance from a given point **iv**
   c. the distance between the center of a circle and any point on the circle **i**
   d. a chord that passes through the center of a circle **ii**
   e. a segment whose endpoints are the center and any point on a circle **i**
   f. a chord made up of two collinear radii **ii**
   g. the distance around a circle **v**

   i. radi
   ii. diar
   iii. chor
   iv. circ
   v. circ
3. Which equations correctly express a relationship in a circle? **A, D, G**
   A. $d = 2r$ B. $C = \pi r$ C. $C = 2d$ D. d =
   E. $r = \frac{d}{\pi}$ F. $C = r^2$ G. $C = 2\pi r$ H. d =

Helping You Remember

4. A good way to remember a new geometric term is to relate the word or its part geometric terms you already know. Look up the origins of the two parts of the *diameter* in your dictionary. Explain the meaning of each part and give a term already know that shares the origin of that part. **Sample answer: The firs comes from *dia*, which means *across* or *through*, as in *diagonal*. second part comes from *metron*, which means *measure*, as in *ge***

Enrichment, p. 546

The Four Color Problem

Mapmakers have long believed that only four colors are necessary to distinguish among any number of different countries on a plane map. Countries that meet only at a point may have the same color provided they do not have an actual border. The conjecture that four colors are sufficient for every conceivable plane map eventually attracted the attention of mathematicians and became known as the "four-color problem." Despite extraordinary efforts over many years to solve the problem, no definite answer was obtained until the 1980s. Four colors are indeed sufficient, and the proof was accomplished by making ingenious use of computers.

The following problems will help you appreciate some of the complexities of the four-color problem. For these "maps," assume that each closed region is a different country.

1. What is the minimum number of colors necessary for each map?
a. b. c.

4 Assess

Open-Ended Assessment

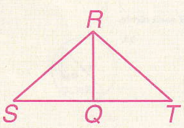
Speaking Students can practice the vocabulary terms in this lesson by describing selected circles and defining terms aloud. For example, find a circle in the lesson without values, and call on students to name its parts. Then ask students to state the values for the radius and circumference of the circle if the diameter is 10 units, 20 units, etc.

Getting Ready for Lesson 10-2

Prerequisite Skill Students will learn about angles and arcs in Lesson 10-2. They will use angle addition to find angle measures in circles. Use Exercises 75–80 to determine your students' familiarity with angle addition.

Answers

73. Given: $\overline{RQ}$ bisects $\angle SRT$.
Prove: $m\angle SQR > m\angle SRQ$



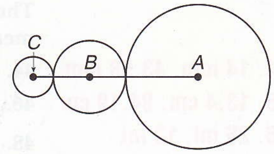
Proof:

Statements (Reasons)

1. $\overline{RQ}$ bisects $\angle SRT$. (Given)
2. $\angle SRQ \cong \angle QRT$ (Def. of $\angle$ bisector)
3. $m\angle SRQ = m\angle QRT$ (Def. of $\cong$ $\angle$s)
4. $m\angle SQR = m\angle T + m\angle QRT$ (Exterior Angle Theorem)
5. $m\angle SQR > m\angle QRT$ (Def. of Inequality)
6. $m\angle SQR > m\angle SRQ$ (Substitution)

FCAT Practice
Standardized Test Practice
A B C D

63. **GRID IN** In the figure, the radius of circle A is twice the radius of circle B and four times the radius of circle C. If the sum of the circumferences of the three circles is 42π, find the measure of $\overline{AC}$. **27**

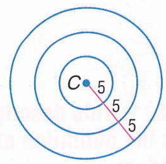


64. **ALGEBRA** There are k gallons of gasoline available to fill a tank. After d gallons have been pumped, what percent of gasoline, in terms of k and d, has been pumped? **A**

Ⓐ $\frac{100d}{k}\%$ Ⓑ $\frac{k}{100d}\%$ Ⓒ $\frac{100k}{d}\%$ Ⓓ $\frac{100k-d}{k}\%$

Extending the Lesson

65. **CONCENTRIC CIRCLES** Circles that have the same center, but different radii, are called *concentric circles*. Use the figure at the right to find the exact circumference of each circle. List the circumferences in order from least to greatest. **10π, 20π, 30π**



Maintain Your Skills

Mixed Review

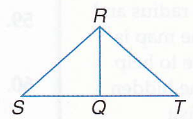
Find the magnitude to the nearest tenth and direction to the nearest degree of each vector. *(Lesson 9-6)*

66. $\overrightarrow{AB} = \langle 1, 4 \rangle$ **4.1; 76°**
67. $\vec{v} = \langle 4, 9 \rangle$ **9.8; 66°**
68. $\overrightarrow{AB}$ if $A(4, 2)$ and $B(7, 22)$ **20.2; 81°**
69. $\overrightarrow{CD}$ if $C(0, -20)$ and $D(40, 0)$ **44.7; 27°**

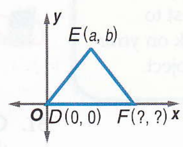
Find the measure of the dilation image of $\overline{AB}$ for each scale factor k. *(Lesson 9-5)*

70. $AB = 5, k = 6$ **30**
71. $AB = 16, k = 1.5$ **24**
72. $AB = \frac{2}{3}, k = -\frac{1}{2}$ **$\frac{1}{3}$**

73. **PROOF** Write a two-column proof. *(Lesson 5-3)*
Given: $\overline{RQ}$ bisects $\angle SRT$.
Prove: $m\angle SQR > m\angle SRQ$
See margin.



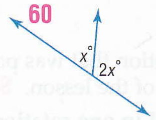
74. **COORDINATE GEOMETRY** Name the missing coordinates if $\triangle DEF$ is isosceles with vertex angle E. *(Lesson 4-3)* **(2a, 0)**



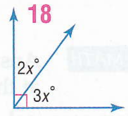
Getting Ready for the Next Lesson

PREREQUISITE SKILL Find x. *(To review **angle addition**, see Lesson 1-4.)*

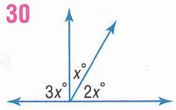
75. **60**



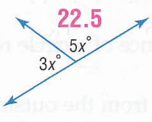
76. **18**



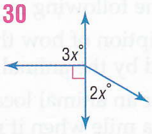
77. **30**



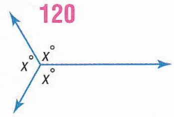
78. **22.5**



79. **30**



80. **120**



Teacher to Teacher

Kim A. Halvorson, DeSoto County High School — Arcadia, FL

My students are asked to decorate a T-shirt with a "pi" theme. Then they wear them on March 14 (3.14). The rest of the school (via morning announcements) is encouraged to ask the geometry students to discuss their shirts.