

Solve the system by graphing.

$$\begin{cases} x + 3y = 2 \\ 3x + 3y = -6 \end{cases} \quad (-4, 2)$$

The table shows the number of pairs of shoes sold by two new employees at a shoe store. Find linear models for each employee's sales. Use the graph of the models to predict the week in which they could sell the same number of pairs of shoes.

Week	1	2	3	4
Ed	50	55	63	67
Jo	40	47	56	62

$$\begin{aligned} \text{Ed: } y &= 5.9x + 44; \\ \text{Jo: } y &= 7.5x + 32.5; \end{aligned}$$

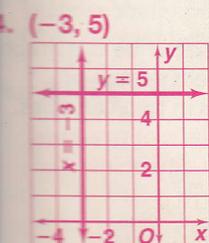
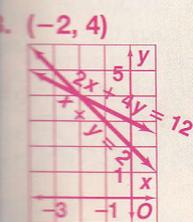
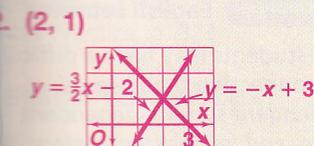
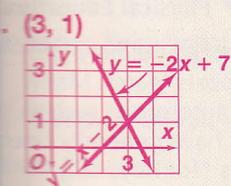
During week 8

Classify the system without graphing.

$$\begin{cases} y = 3x + 2 \\ -6x + 2y = 4 \end{cases}$$

dependent system

Pages 118–121 Exercises



Reading Math
Unique means "exactly one."

You can classify a system of two linear equations by the number of solutions. A system that has a unique solution, as in Examples 1 and 2, is an **independent system**. However, not every system has a unique solution.

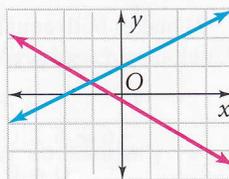
A **dependent system** does not have a unique solution. An **inconsistent system** is a system that has no solution.

Key Concepts

Summary

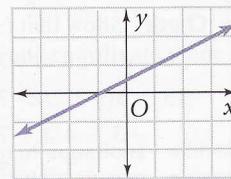
Graphical Solutions of Linear Systems in Two Variables

Intersecting Lines



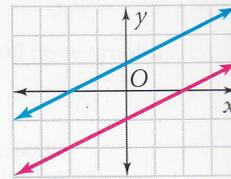
one solution
Independent

Coinciding Lines



no unique solution
Dependent

Parallel Lines



no solution
Inconsistent

You can also classify a system of equations without graphing. By comparing the slopes and y-intercepts of the equations, you can find the number of solutions.

3 EXAMPLE Classifying Systems Without Graphing

Classify the system without graphing. $\begin{cases} y = 2x + 3 \\ -2x + y = 1 \end{cases}$

$$\begin{aligned} y &= 2x + 3 && \text{Rewrite in slope-intercept form.} \rightarrow y = 2x + 1 \\ m &= 2, b = 3 && \leftarrow \text{Find the slope and y-intercept.} \rightarrow m = 2, b = 1 \end{aligned}$$

Since the slopes are the same, the lines could coincide. Compare the y-intercepts. Since the y-intercepts are different, the lines are parallel. There is no solution. The system is an inconsistent system.

Check Understanding 3 Without graphing, classify each system as *independent*, *dependent*, or *inconsistent*.

$$\begin{aligned} \text{a. } \begin{cases} 3x + y = 5 \\ 15x + 5y = 2 \end{cases} && \text{inconsistent} && \text{b. } \begin{cases} y = 2x + 3 \\ -4x + 2y = 6 \end{cases} && \text{dependent} \\ \text{c. } \begin{cases} x - y = 5 \\ y + 3 = 2x \end{cases} && \text{independent} \end{aligned}$$

EXERCISES

For more practice, see *Extra Practice*.

Practice and Problem Solving

A Practice by Example Solve each system by graphing. Check your answers. 1–5. See margin.

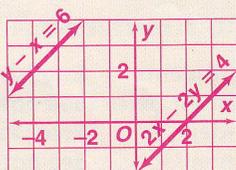
Example 1
(page 117)

- $\begin{cases} y = x - 2 \\ y = -2x + 7 \end{cases}$
- $\begin{cases} y = -x + 3 \\ y = \frac{3}{2}x - 2 \end{cases}$
- $\begin{cases} 2x + 4y = 12 \\ x + y = 2 \end{cases}$
- $\begin{cases} x = -3 \\ y = 5 \end{cases}$
- $\begin{cases} 2x - 2y = 4 \\ y - x = 6 \end{cases}$
- $\begin{cases} 3x + y = 5 \\ x - y = 7 \end{cases}$
- $\begin{cases} -5x + y = -9 \\ x + 3y = 21 \end{cases}$
- $\begin{cases} y = x \\ y - 5x = 0 \end{cases}$
- $\begin{cases} x = 10 \\ x = y - 10 \end{cases}$

6–9. See back of book.

118 Chapter 3 Linear Systems

5. no solution



12a. Answers may vary.

$$\begin{cases} y = 3000x + 5200 \\ y = -900x + 35,700 \end{cases}$$

b. If Feb = 1, the revenue will equal expenses in the 7.82 month, or August.

Example 2 (page 117)

For Exercises 10–11, use your graphing calculator. Find linear models for each set of data. Use each model to predict the year in which the quantities will be equal.

$$y = 0.174x + 0.1$$

$$y = 0.1107x + 2.354$$

about 2005

$$y = 0.2182x + 67.52$$

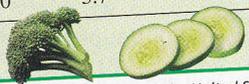
$$y = 0.1545x + 75.463$$

about 2095

10. Annual U.S. Consumption of Vegetables

Year	Broccoli (lb/person)	Cucumbers (lb/person)
1970	0.5	2.6
1975	1.0	2.6
1980	1.4	3.6
1985	2.6	4.0
1990	3.4	4.3
1995	4.4	5.2
2000	5.7	5.8

SOURCE: Statistical Abstract of the United States. Go to www.PHSchool.com for a data update. Web Code: agg-2041



11. U.S. Life Expectancy at Birth

Year	Men (years)	Women (years)
1970	67.1	74.7
1975	68.8	76.6
1980	70.0	77.4
1985	71.1	78.2
1990	71.8	78.8
1995	72.5	78.9
1997	73.6	79.4

SOURCE: U.S. Census Bureau. Go to www.PHSchool.com for a data update. Web Code: agg-2041



12. a. **Business** The spreadsheet shows the monthly revenue and monthly expenses for a new business. Find a linear model for monthly revenue and a linear model for monthly expenses. **a–b. See margin p. 118.**
 b. Use the models to predict the month in which revenue will equal expenses.

	A	B	C
1	Month	Revenue	Expenses
2	Feb	8000	35000
3	Mar	12000	33000
4	Apr	13000	34000
5	May	18000	32000
6	Jun	20000	31000

Example 3 (page 118)

Without graphing, classify each system as independent, dependent, or inconsistent.

13. $\begin{cases} 7x - y = 6 \\ -7x + y = -6 \end{cases}$ **dependent**
14. $\begin{cases} -3x + y = 4 \\ x - \frac{1}{3}y = 1 \end{cases}$ **inconsistent**
15. $\begin{cases} 4x + 8y = 12 \\ x + 2y = -3 \end{cases}$ **inconsistent**
16. $\begin{cases} y = 2x - 1 \\ y = -2x + 5 \end{cases}$ **independent**
17. $\begin{cases} x = 6 \\ x = -2 \end{cases}$ **inconsistent**
18. $\begin{cases} 2y = 5x + 6 \\ -10x + 4y = 8 \end{cases}$ **inconsistent**
19. $\begin{cases} x - 3y = 2 \\ 4x - 12y = 8 \end{cases}$ **dependent**
20. $\begin{cases} x + 4y = 12 \\ 2x - 8y = 4 \end{cases}$ **independent**
21. $\begin{cases} 4x + 8y = -6 \\ 6x + 12y = -9 \end{cases}$ **dependent**
22. $\begin{cases} 4y - 2x = 6 \\ 8y = 4x - 12 \end{cases}$ **inconsistent**
23. $\begin{cases} y - x = 0 \\ y = -x \end{cases}$ **independent**
24. $\begin{cases} 2y - x = 4 \\ \frac{1}{2}x - y = 2 \end{cases}$ **inconsistent**

Graph and solve each system. Where necessary, estimate the solution.

25. $\begin{cases} 3 = 4y + x \\ 4y = -x + 3 \end{cases}$
26. $\begin{cases} x - 2y + 1 = 0 \\ x + 4y - 6 = 0 \end{cases}$
27. $\begin{cases} 3x + 6y - 12 = 0 \\ x + 2y = 8 \end{cases}$
28. $\begin{cases} -x + 3y = 6 \\ 2x - y = 8 \end{cases}$
29. $\begin{cases} 3x + y = 3 \\ 2x - y = 7 \end{cases}$
30. $\begin{cases} 2x + 3y = 6 \\ 4x = 6y + 3 \end{cases}$
31. $\begin{cases} 10 - 3x = -3y \\ 2 = 2x + y \end{cases}$
32. $\begin{cases} 3x = -5y + 4 \\ 250 + 150x = 300 \end{cases}$
33. $\begin{cases} x + 3y = 6 \\ 6y + 2x = 12 \end{cases}$
34. $\begin{cases} 2y + x = 8 \\ y - 2x = -6 \end{cases}$
35. $\begin{cases} y = -2x + 6 \\ x - 3y = -6 \end{cases}$
36. $\begin{cases} -x - 2 = -2y \\ 2x - 4y - 4 = 0 \end{cases}$

Assignment Guide

1 Objective

- A Core 1–50
- C Extension 51–55

Standardized Test Prep 56

Mixed Review 62–74

Exercise 5 If students draw graphs correctly, the lines will look parallel. Be sure students understand how slope can be used to confirm that the lines really are parallel.

Error Prevention

Exercises 28, 29, 33 Students avoid errors by first multiplying both sides of each equation.

Enrichment 3-1

Reteaching 3-1

Practice 3-1

Practice 3-1

Classify each system without graphing.

- $\begin{cases} x + y = 3 \\ y = 2x - 3 \end{cases}$
- $\begin{cases} 2x + y = 7 \\ x - 2y = 1 \end{cases}$
- $\begin{cases} x + y = 4 \\ y = 2x + 1 \end{cases}$
- $\begin{cases} 3x + 2y = 7 \\ 3x - 5y = -9 \end{cases}$
- $\begin{cases} x + y = 3 \\ y = 2x - 3 \end{cases}$
- $\begin{cases} 2x + y = 7 \\ x - 2y = 1 \end{cases}$
- $\begin{cases} x + y = 3 \\ y = 2x - 3 \end{cases}$
- $\begin{cases} 2x + y = 7 \\ x - 2y = 1 \end{cases}$
- $\begin{cases} x + y = 3 \\ y = 2x - 3 \end{cases}$
- $\begin{cases} 2x + y = 7 \\ x - 2y = 1 \end{cases}$

13. The spreadsheet below shows the monthly revenue and monthly expenses for a new business. Find a linear model for monthly revenue and a linear model for monthly expenses. Use the models to estimate the month in which revenue will equal expenses.

	A	B	C
1	Month	Revenue	Expenses
2	Feb	8000	35000
3	Mar	12000	33000
4	Apr	13000	34000
5	May	18000	32000
6	Jun	20000	31000

Solve each system by graphing. Check your answer.

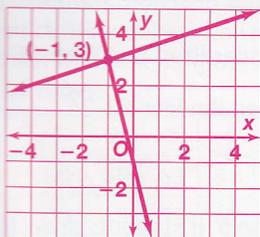
- $\begin{cases} y = x - 2 \\ x + y = 10 \end{cases}$
- $\begin{cases} 2x + y = 11 \\ x - 2y = 4 \end{cases}$
- $\begin{cases} x + y = 16 \\ x + y = 7 \end{cases}$
- $\begin{cases} x + y = 16 \\ x + y = 7 \end{cases}$
- $\begin{cases} x + y = 16 \\ x + y = 7 \end{cases}$
- $\begin{cases} x + y = 16 \\ x + y = 7 \end{cases}$

Lesson 3-1 Practice

Lesson Quiz 3-1

1. Graph and solve the system.

$$\begin{cases} 4x + y = -1 \\ -x + 3y = 10 \end{cases} \quad (-1, 3)$$



Classify each system without graphing. Tell how many solutions there are.

2. $\begin{cases} 5x + 3y = 10 \\ -x - 0.6y = -2 \end{cases}$

dependent; infinitely many

3. $\begin{cases} 12x - 18y = 9 \\ -6x + 9y = 13 \end{cases}$

inconsistent; no solutions

4. $\begin{cases} 4x + 5y = -10 \\ 3x - 8y = 15 \end{cases}$

independent; one solution

Alternative Assessment

Have students work in groups of three. Students 1 and 2 each write an equation of the form $Ax + By = C$, where A , B , and C are non-zero integers from -9 to 9 . Student 3 classifies and graphs the system. If there is a unique solution with integer x - and y -values, student 3 identifies the solution and checks that it is correct. Otherwise, student 3 estimates the solution. Students 1 and 2 then confirm the correctness of the work. Students change roles until each student has graphed two systems.

Exercise 53 Check that students understand what the solution set notation means geometrically.



Real-World Connection

Groomers must be able to handle dogs of every breed and temperament.

Challenge

53. Answers may vary.

Sample:

$$\begin{cases} -10x + 2y = 4 \\ 5x - y = -2 \end{cases}$$

37. **Banking** To pay your monthly bills, you can either open a checking account or use an online banking service. A local bank charges \$3 per month and \$0.40 per check, while an online services charges a flat fee of \$9 per month.
- Write and graph a system of linear equations to model the cost c of each service for b bills that you need to pay monthly. **See back of book.**
 - Find the point of intersection of the two linear models. What does this point represent? **b-c. See margin.**
 - If you pay about 12 bills per month, which service should you choose? Explain.

Classify each system without graphing.

38. $\begin{cases} 3x - 2y = 8 \\ 4y = 6x - 5 \end{cases}$ **inconsistent**

39. $\begin{cases} 2x + 8y = 6 \\ x = -4y + 3 \end{cases}$ **dependent**

40. $\begin{cases} 3a + 6b = 14 \\ -a + 2b = 3 \end{cases}$ **independent**

41. $\begin{cases} 3m = -5n + 4 \\ n - \frac{6}{5} = -\frac{3}{5}m \end{cases}$ **inconsistent**

42. $\begin{cases} -12x + 4y = 8 \\ y - 4 = 3x \end{cases}$ **inconsistent**

43. $\begin{cases} -6y + 18 = 9 \\ 3y + 6x = 9 \end{cases}$ **dependent**

44. **Fees** Suppose you are going on vacation and leaving your dog in a kennel. Bowowery charges \$25 per day, which includes a one-time grooming treat. The Poochpad charges \$20 per day and a one-time fee of \$30 for grooming.
- Write a system of equations to represent the cost c for d days that your dog will stay at a kennel. **a, c. See margin p. 121.**
 - Using a graphing calculator, find the number of days for which the cost will be the same. **See back of book.**
 - If your vacation is a week long, which kennel should you choose? Explain.
45. **Advertising** You and your business partner are mailing advertising flyers to your customers. You address 6 flyers each minute and have already done 100. Your partner addresses 4 flyers each minute and has already done 100. Graph and solve a system of equations to find when the two of you will have addressed equal numbers of flyers. **See back of book.**

Open-Ended Write a second equation for each system so that the system will have the indicated number of solutions. **46–48. Answers may vary. Samples:**

46. one
 $\begin{cases} y = -3x + 2 \\ \underline{\hspace{1cm}} \end{cases}$
 $y = x + 3$

47. none
 $\begin{cases} y = -4x - 6 \\ \underline{\hspace{1cm}} \end{cases}$
 $y = -4x + 8$

48. an infinite number
 $\begin{cases} 3y = 6x + 7 \\ \underline{\hspace{1cm}} \end{cases}$
 $y = 2x + \frac{7}{3}$

49. **Reasoning** Is it possible for an inconsistent linear system to consist of two equations with the same y -intercept? Explain. **No; they would be the same line.**
50. **Writing** Summarize the possible relationships for the y -intercepts, slopes, and number of solutions in a system of two linear equations of two variables. **See margin.**

Open-Ended Write a second equation for each system so that the system will have the indicated number of solutions. **51–52. Answers may vary. Samples:**

51. infinite number of solutions
 $\begin{cases} \frac{x}{4} + \frac{y}{3} = 1 \\ \underline{\hspace{1cm}} \end{cases}$ **$3x + 4y = 12$**

52. no solutions
 $\begin{cases} 5x + 2y = 10 \\ \underline{\hspace{1cm}} \end{cases}$ **$y = -\frac{5x}{2} + 7$**

53. Write a system of linear equations with the solution set $\{(x, y) \mid y = 5x - 10\}$.
54. **Critical Thinking** Look back through the exercises on the previous two pages to find several dependent systems. What relationship exists between the equations in each system? **They are the same equation written in different forms.**

pages 118–121 Exercises

37b. (15, 9); the point represents where the cost of using the bank or online service would be the same.

c. The local bank would be cheaper if you only have 12 bills to pay per month.

Reading Math
A widget is a small, unspecified gadget.

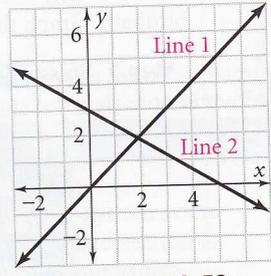
55. **Economics** Research shows that in a certain market only 2000 widgets can be sold at \$8 each, but if the price is reduced to \$3, then 10,000 can be sold.
- Let p represent price and n represent the number of widgets. Identify the independent variable and the dependent variable.
 - Use the information above to write a linear *demand* equation.
 - A shop can make 2000 widgets for \$5 each and 20,000 widgets for \$2 each. Use this information to write a linear *supply* equation.
 - Find the equilibrium point where supply is equal to demand and profit is a maximum. Explain the meaning of the coordinates of this point within the context of the exercise.
- a. p : independent, n : dependent**
b. $n = -1600p + 14,800$
c. $n = -6000p + 32,000$
- d. About (3.91, 8545); profits are maximized if about 8545 widgets are sold for about \$3.91 each.**

FCAT Practice

Multiple Choice

56. Which is an equation for Line 2? **C**
 A. $3x - 5y = 15$
 B. $3x + 5y = 3$
 C. $3x + 5y = 15$
 D. $5x + 3y = 15$
57. Which is NOT an equation for Line 1? **G**
 F. $y = x$
 G. $x + y = 0$
 H. $x - y = 0$
 I. $y - x = 0$
58. Which point lies on both Line 1 and Line 2? **B**
 A. (0, 0)
 B. (1.875, 1.875)
 C. (1.95, 1.95)
 D. (2, 2)
59. What is the solution of the system?

$$\begin{cases} 5x + 6y = -24 \\ -2x + 3y = 15 \end{cases}$$
 H
 F. (6, -1) G. (6, 1) H. (-6, 1) I. (-6, -1)



Exercises 56–58.

FCAT Online
FCAT Format quiz at www.PHSchool.com
Web Code: aga-0301

Short Response

60. Explain how you can use slopes to show that the system $\begin{cases} 2x - 5y = 23 \\ 3y - 7x = -8 \end{cases}$ is NOT inconsistent. **60–61. See margin.**

Extended Response

61. One equation of a system of equations is $2x - 3y = 5$.
- Find a second equation such that the system is dependent.
 - Find a second equation such that the system is inconsistent.

Mixed Review

Lesson 2–7

Graph each inequality on a coordinate plane. **62–64. See back of book.**

62. $3x - 4y \geq 16$ 63. $-5x > 8y + 4$ 64. $x < -4$

Lesson 2–2

Write an equation for each line.

65. $m = -\frac{2}{3}$; contains $(-9, 4)$ **$y = -\frac{2}{3}x - 2$** 66. $m = 0$; contains $(3, 4)$ **$y = 4$**
 67. $m = 2$; contains $(-2, -3)$ **$y = 2x + 1$** 68. $m = -\frac{1}{2}$; contains $(2, -6)$ **$y = -\frac{1}{2}x - 5$**

Lesson 1–3

Solve each equation and check the solution.

69. $3n = -4(2 + n) - \frac{8}{7}$ 70. $-4a + a = 7a - 6 - \frac{3}{5}$ 71. $\frac{x}{3} + 5 = \frac{1}{6} - 14.5$
 72. $4x - 2 = \frac{1}{2}x + \frac{4}{7}$ 73. $\frac{r}{5} + 5 = r - 3$ **10** 74. $2(m - 3) = -4$ **1**

Resources

- For additional practice with variety of test item formats
- FCAT Practice, p. 161
 - FCAT Strategies, p. 156
 - FCAT Daily Practice and Strategies Transparencie

Exercise 57 Write each equation in slope-intercept form. Then identify the equation that is the same as the other three.

Exercise 59 If an answer choice fails for one of the equations, you know that it is not the solution to the system. Only test an answer choice in both equations if it satisfies the first equation.

60. [2] (The slope of $2x - 5y = 23$ is $\frac{2}{5}$, the slope of $3y - 7x = -8$ is $\frac{7}{3}$. Since the slopes are not equal, the lines are not parallel; they do not coincide. So the lines intersect; the system has exactly one solution and is consistent.)
 [1] does not include explanation

61. [4] (a) A second equation is $4x - 6y = 10$ or any equation of the form $2ax - 3ay = 5$ where $a \neq 0$.
 (b) A second equation is $2x - 3y = 5$ or any equation of the form $2ax - 3ay = 5$ where $a \neq 0$.
 [3] minor error in part (a) or (b)
 [2] minor error in parts (a) and (b)
 [1] only complete (a) or (b)

$c = 20d + 30$
 $c = 25d$

The Pooch Pad would be cheaper for a 7-day stay.

50. An independent system has one solution. The slopes are different, but the y -intercepts could be the same. An inconsistent system has no solution. The slopes

are the same, and the y -intercept are different. A dependent system has an infinite number of solutions. The slopes and y -intercepts are the same.