5.6/5.7 Factoring Polynomials I

1st Step always is to factor a GCF (greatest common factor) if possible: (factor numbers and variable)

$$12x + 20 = 4(3x+5)$$

$$a^3 + 6a^2 - 11a = a(a^2+6a-11)$$

$$-a^3 + 6a^2 - 11a = -a(a^2-6a+11)$$

2nd Step is to factor what's left.

Factoring:

1) Always look at your "b" and "c":

a) If "b" and "c" are positive than the factors of "c" are positive and the operations in the binomial are both addition.

b) If "b" is negative and "c" is positive then the factors of "c" are negative and the operations in the binomial are both subtraction.

c) If "b" is positive or negative and "c" is negative then the factors of "c" are negative and one operation is addition and one is subtraction.

*** a) through c) are extremely helpful ***

2) See if any variable and/or # is a Greatest Common Factor of ALL the terms

3) Simplify into 2 binomials.

2nd step is to factor what is left:
Example:

x^2 + 8x +15 There is no GCF to factor.
Your "b" and "c" are both positive.

Step 1: Identify the values of a, b and c
a = 1 b = 8 c = 15

Step 2:
In separate columns - List "a" twice , next put "+/- = b" and then "a * c"

Step 3: *** Look at "b" and "a *c". "a * c" is the result of multiplication. "b" is the result of addition/subtraction SO:

Find the factors of "a * c" that multiply to 15 and add to 8. This would be 3 and 5

Step 4: Put the factors under the two "a"s. Reduce if possible

Step 5: Place the "fractions" into the binomials in the same order.

Check it by using FOIL on your answer. It better equal your original trinomial.

$x^2+8x+15$

① $a=1 \quad b=8 \quad c=15$

② $\frac{*1x}{5} \quad \frac{*1x}{3}$ add to $b=8$ $a*c=15$

$5+3$ $\quad 5\cdot 3$

*Write "a" with its variable 2 times

$(x+5)(x+3)$

@ $x^2+4x+3 = (+)(+)$

ⓑ $x^2-4x+3 = (-)(-)$

C_1 $x^2-2x-3 = (+)(-)$

C_2 x^2+2x-3 $(+)(-)$

$x^2 + 4x + 3$

① $a = 1$, $b = 4$, $c = 3$

② Write "a" twice w/ variable

$\frac{1x}{1}$ $\frac{3x}{3}$ add to $b=4$: $1+3$ $a \cdot c = 3$: $1 \cdot 3$

$(x+1)(x+3)$

$x^2 - 4x + 3$

$a = 1$
$b = -4$
$c = 3$

$1x \quad 1x$
$-3 \quad -1$

add to $b = -4$
$-3 + -1$

$a \cdot c = 3$
$-3 \cdot -1$

$(x-3)(x-1)$

$x^2 - 2x - 3$

$a = 1$
$b = -2$
$c = -3$

$1x \quad 1x$
$1 \quad -3$

$(x+1)(x-3)$

add to $b = -2$

~~$-1+3=2$~~

$1 + -3 = -2$

$a \cdot c = -3$

$-1 \cdot 3$
$1 \cdot -3$

x^2+2x-3

$a=1$

$b=2$

$c=-3$

$\underline{x} \quad \underline{x}$

$-1 \quad 3$

add to $b=2$

$-1+3=2$

$a \cdot c = -3$

$-1 \cdot 3$

$1+-3=-2$ ~~$1 \cdot -3$~~

$(x-1)(x+3)$

$x^2 + 13x + 40$

$a = 1$
$b = 13$
$c = 40$

$a \cdot c$ multiply 40

$8 \cdot 5$

add to $b = 13$

$8 + 5$

$7x \quad 2x$

$8 \quad 5$

$(x+8)(x+5)$

$x^2+3x-18$

$a=1$
$b=3$
$c=-18$

x x add $b=3$ $a \cdot c = -18$

6 -3 ~~$+ + -18 = -17$~~ ~~$1 \cdot -18$~~

$6+-3$ $6 \cdot -3$

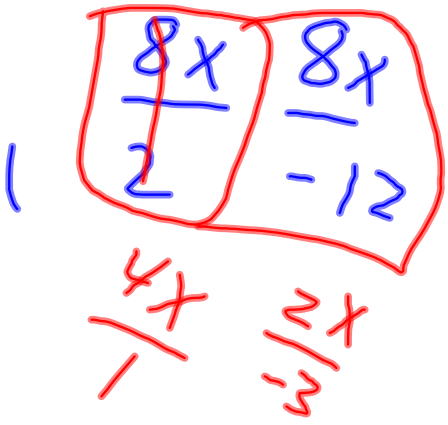
$(x+6)(x-3)$

$8x^2 - 10x - 3$

$a = 8$

$b = -10$

$c = -3$



add to $b = -10$

$2 + -12$

$a \cdot c = -24$

$2 \cdot -12$

$4x$ 1

$2x$ -3

$(4x + 1)(2x - 3)$

$6x^2 - 7x - 10$

$a = 6$
$b = -7$
$c = -10$

$6x \quad 6x$

add to $b = -7$

$a \cdot c = -60$

$-12 \quad 5$

$-12 + 5$

$-12 \cdot 5$

$\frac{1x}{-2} \quad \frac{6x}{5}$

$(x - 2)(6x + 5)$

P.66 Factoring a trinomial with Two variables

$2x^2 - 7xy + 3y^2$

$a=2 \quad b=-7 \quad c=3$

* Solve as if the 2nd variable is not there

$2x^2 - 7x + 3$

$2x \cdot 2x$ $\quad -6 \cdot -1$ $\quad a \cdot c = 6$

add $b=7$ $\quad -6 + -1$ $\quad -6 \cdot 1$

$x \quad 2x$

$-3 \quad -1$

$(x-3y)(2x-1y)$

* now add in the 2nd variable

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

$11^2 = 121$

$12^2 = 144$

$13^2 = 169$

Perfect square trinomials (p. 67-68)

* If the first and last terms are perfect squares then shortcuts apply (You can always still factor the regular way):

$9x^2 - 121 = (3x + 11)(3x - 11)$

no middle term

$$9x^2 - 42x + 49$$

"a" and "c" are perfect squares

$$(3x - 7)(3x - 7)$$

$$x^4 - 81 = (x^2+9)(x^2-9)$$
$$= (x^2+9)(x+3)(x-3)$$

x^2+6x+9 1 and 9 are perfect squares

Look at signs so answer is:

$(+)(+)$ Put in perfect squares

$(1x+3)(1x+3)$

$x^2 - 12x + 36$